K–12 Mathematics TEKS

2009 Update (1997 Adoption)
and 2012 Adoption

Texas Essential Knowledge and Skills

Printed and distributed by the Charles A. Dana Center at The University of Texas at Austin

Texas Administrative Code (TAC), Title 19, Part 2, Chapter 111, Texas Essential Knowledge and Skills for Mathematics (Subchapters A through D)

Includes the English Language Proficiency Standards
Mathematics
Texas Essential Knowledge and Skills

For kindergarten through grade 12
Texas Administrative Code (TAC), Title 19, Part 2
Chapter 111. Texas Essential Knowledge and Skills for Mathematics
(Subchapters A through D)

Including the
English Language Proficiency Standards
Texas Administrative Code (TAC), Title 19, Part 2
Chapter 74, Curriculum Requirements; Subchapter A, Required Curriculum;
Rule §74.4, English Language Proficiency Standards

About the Mathematics Texas Essential Knowledge and Skills
The mathematics Texas Essential Knowledge and Skills (TEKS) were developed by the state of Texas to clarify what all students should know and be able to do in mathematics in kindergarten through grade 12.

Texas school districts are required to provide instruction in the mathematics TEKS, which were originally adopted by the State Board of Education in 1997 and implemented statewide in 1998. The first revision of the mathematics TEKS was adopted by the State Board of Education in 2005 and implemented statewide in 2006–2007. The second revision of the mathematics TEKS was adopted by the State Board of Education in 2009.

In 2012, the State Board of Education adopted new mathematics TEKS to be implemented beginning in 2014–2015 in kindergarten through grade 8, and beginning in 2015–2016 in high school.

Both the 2009 and the 2012 versions of the mathematics TEKS are published in this book.

About the English Language Proficiency Standards
The English Language Proficiency Standards are part of Texas state law. The Charles A. Dana Center is providing printed copies of these standards as a public service. You may also access the ELPS free online via the Texas Secretary of State’s online Texas Administrative Code, which may be found at www.sos.state.tx.us/tac/index.shtml (scroll to the bottom of the page and click on this link: http://info.sos.state.tx.us/pls/pub/readtac$ext.viewtac.

According to the TAC administrative rule cited in (a) Introduction to the ELPS,

“(1) The English language proficiency standards … outline English language proficiency level descriptors and student expectations for English language learners (ELLs). School districts shall implement this section as an integral part of each subject in the required curriculum. The English language proficiency standards are to be published along with the Texas Essential Knowledge and Skills (TEKS) for each subject in the required curriculum.”
About the development of this resource

The Dana Center is printing these mathematics TEKS—along with the English Language Proficiency Standards—as a service to educators in Texas who want a bound version of the standards. In keeping with our longstanding practice, we use all funds generated through materials we publish to further our nonprofit educational mission.

This booklet is intended to help educators familiarize themselves with the latest version of the mathematics TEKS and to use these TEKS, in conjunction with the ELPS, to plan instruction and assessment.

About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center works at scale with our nation’s education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy.

We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.

Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement. We help our partners adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at www.utdanacenter.org.

We use all funds generated through use of our materials to further our nonprofit mission. Please send your permission requests or questions to us at this address:

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Resources

The TEKS for mathematics, as well as for other subject areas, can be downloaded in printable format, free of charge, via the Texas Education Agency website, [www.tea.state.tx.us](http://www.tea.state.tx.us). Bound versions of the mathematics TEKS can be ordered from the Dana Center product catalog at [www.utdanacenter.org/catalog](http://www.utdanacenter.org/catalog) or by contacting the Dana Center at 1-866-871-9995.

The Dana Center also provides resources, including professional development, for implementing the state standards. See [www.utdanacenter.org/catalog](http://www.utdanacenter.org/catalog) for our products and [www.utdanacenter.org/pd](http://www.utdanacenter.org/pd) to sign up for our professional development. You may also find out about professional development opportunities by calling 512-471-6190.

For more information about the Dana Center and our programs and resources, see our homepage at [www.utdanacenter.org](http://www.utdanacenter.org).

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About the finding aids we have added to this version of the mathematics TEKS

We have made a few small changes to the framing and formatting of the mathematics TEKS as they appear online in the Texas Administrative Code, to make it easier for readers to navigate around the K-12 TEKS. Specifically, we have added

- Running footers (on every page of this version of the mathematics TEKS proper) that specify “Chapter 111. Texas Essential Knowledge and Skills for Mathematics” and “2009 Update”

- Running headers that specify that these are the Revised 2009 version of the mathematics TEKS and that note the grade level (K-8) or course (high school).

- Additional lettering or numbering to the Knowledge and Skills statements. For example, the 2009 mathematics Knowledge and Skills statements in the online version of the TAC available on the Texas Secretary of State’s website

  In kindergarten, are numbered (1) through (15); in grade 1, are numbered (1) through (13), and so on. In contrast, in our layout of the mathematics TEKS, we have numbered the kindergarten Knowledge and Skills statements (K.1) through (K.15); the grade 1 K&S statements, (1.1) through (1.13), and so on up through the grades.

  Similarly, in high school, the K&S statements in the TAC online in Algebra I are numbered (1) through (11); in Algebra II, (1) through (11); in Precalculus, (1) through (6), and so on through the courses. In contrast, in our layout of the mathematics TEKS, we have numbered the Algebra I K&S statements (A.1) through (A.11), the Algebra II K&S statements (2A.1) through (2A.11), the Precalculus K&S statements (P.1) through (P.6), and so on.
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2009 Update (1997 Adoption)

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The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year.


(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Kindergarten are developing whole-number concepts and using patterns and sorting to explore number, data, and shape.

(2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.

(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(K.1) Number, operation, and quantitative reasoning. The student uses numbers to name quantities.

The student is expected to:

(A) use one-to-one correspondence and language such as more than, same number as, or two less than to describe relative sizes of sets of concrete objects;

(B) use sets of concrete objects to represent quantities given in verbal or written form (through 20); and

(C) use numbers to describe how many objects are in a set (through 20) using verbal and symbolic descriptions.
Ch. 111, TEKS for Mathematics (Revised 2009)

(K.2) **Number, operation, and quantitative reasoning.** The student describes order of events or objects.

The student is expected to:

(A) use language such as before or after to describe relative position in a sequence of events or objects; and

(B) name the ordinal positions in a sequence such as first, second, third, etc.

(K.3) **Number, operation, and quantitative reasoning.** The student recognizes that there are quantities less than a whole.

The student is expected to:

(A) share a whole by separating it into two equal parts; and

(B) explain why a given part is half of the whole.

(K.4) **Number, operation, and quantitative reasoning.** The student models addition (joining) and subtraction (separating).

The student is expected to model and create addition and subtraction problems in real situations with concrete objects.

(K.5) **Patterns, relationships, and algebraic thinking.** The student identifies, extends, and creates patterns.

The student is expected to identify, extend, and create patterns of sounds, physical movement, and concrete objects.

(K.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to make predictions.

The student is expected to:

(A) use patterns to predict what comes next, including cause-and-effect relationships; and

(B) count by ones to 100.

(K.7) **Geometry and spatial reasoning.** The student describes the relative positions of objects.

The student is expected to:

(A) describe one object in relation to another using informal language such as over, under, above, and below; and

(B) place an object in a specified position.

(K.8) **Geometry and spatial reasoning.** The student uses attributes to determine how objects are alike and different.

The student is expected to:

(A) describe and identify an object by its attributes using informal language;

(B) compare two objects based on their attributes; and

(C) sort a variety of objects including two- and three-dimensional geometric figures according to their attributes and describe how the objects are sorted.
(K.9) **Geometry and spatial reasoning.** The student recognizes attributes of two- and three-dimensional geometric figures.

The student is expected to:

- (A) describe and compare the attributes of real-life objects such as balls, boxes, cans, and cones or models of three-dimensional geometric figures;
- (B) recognize shapes in real-life three-dimensional geometric figures or models of three-dimensional geometric figures; and
- (C) describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle).

(K.10) **Measurement.** The student directly compares the attributes of length, area, weight/mass, capacity, and/or relative temperature. The student uses comparative language to solve problems and answer questions.

The student is expected to:

- (A) compare and order two or three concrete objects according to length (longer/shorter than, or the same);
- (B) compare the areas of two flat surfaces of two-dimensional figures (covers more, covers less, or covers the same);
- (C) compare two containers according to capacity (holds more, holds less, or holds the same);
- (D) compare two objects according to weight/mass (heavier than, lighter than or equal to); and
- (E) compare situations or objects according to relative temperature (hotter/colder than, or the same as).

(K.11) **Measurement.** The student uses time to describe, compare, and order events and situations.

The student is expected to:

- (A) compare events according to duration such as more time than or less time than;
- (B) sequence events (up to three); and
- (C) read a calendar using days, weeks, and months.

(K.12) **Probability and statistics.** The student constructs and uses graphs of real objects or pictures to answer questions.

The student is expected to:

- (A) construct graphs using real objects or pictures in order to answer questions; and
- (B) use information from a graph of real objects or pictures in order to answer questions.
(K.13) **Underlying processes and mathematical tools.** The student applies Kindergarten mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify mathematics in everyday situations;
(B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
(D) use tools such as real objects, manipulatives, and technology to solve problems.

(K.14) **Underlying processes and mathematical tools.** The student communicates about Kindergarten mathematics using informal language.

The student is expected to:

(A) communicate mathematical ideas using objects, words, pictures, numbers, and technology; and
(B) relate everyday language to mathematical language and symbols.

(K.15) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.
§111.13. Mathematics, Grade 1.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 1 are building number sense through number relationships, adding and subtracting whole numbers, organizing and analyzing data, and working with two- and three-dimensional geometric figures.

(2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.

(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(1.1) Number, operation, and quantitative reasoning. The student uses whole numbers to describe and compare quantities.

The student is expected to:

(A) compare and order whole numbers up to 99 (less than, greater than, or equal to) using sets of concrete objects and pictorial models;

(B) create sets of tens and ones using concrete objects to describe, compare, and order whole numbers;

(C) identify individual coins by name and value and describe relationships among them; and

(D) read and write numbers to 99 to describe sets of concrete objects.

(1.2) Number, operation, and quantitative reasoning. The student uses pairs of whole numbers to describe fractional parts of whole objects or sets of objects.

The student is expected to:

(A) separate a whole into two, three, or four equal parts and use appropriate language to describe the parts such as three out of four equal parts; and

(B) use appropriate language to describe part of a set such as three out of the eight crayons are red.
(1.3) **Number, operation, and quantitative reasoning.** The student recognizes and solves problems in addition and subtraction situations.

The student is expected to:

(A) model and create addition and subtraction problem situations with concrete objects and write corresponding number sentences; and

(B) use concrete and pictorial models to apply basic addition and subtraction facts (up to $9 + 9 = 18$ and $18 - 9 = 9$).

(1.4) **Patterns, relationships, and algebraic thinking.** The student uses repeating patterns and additive patterns to make predictions.

The student is expected to identify, describe, and extend concrete and pictorial patterns in order to make predictions and solve problems.

(1.5) **Patterns, relationships, and algebraic thinking.** The student recognizes patterns in numbers and operations.

The student is expected to:

(A) use patterns to skip count by twos, fives, and tens;

(B) find patterns in numbers, including odd and even;

(C) compare and order whole numbers using place value;

(D) use patterns to develop strategies to solve basic addition and basic subtraction problems; and

(E) identify patterns in related addition and subtraction sentences (fact families for sums to 18) such as $2 + 3 = 5, 3 + 2 = 5, 5 - 2 = 3,$ and $5 - 3 = 2$.

(1.6) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

The student is expected to:

(A) describe and identify two-dimensional geometric figures, including circles, triangles, rectangles, and squares (a special type of rectangle);

(B) describe and identify three-dimensional geometric figures, including spheres, rectangular prisms (including cubes), cylinders, and cones;

(C) describe and identify two- and three-dimensional geometric figures in order to sort them according to a given attribute using informal and formal language; and

(D) use concrete models to combine two-dimensional geometric figures to make new geometric figures.
(1.7) **Measurement.** The student directly compares the attributes of length, area, weight/mass, capacity, and temperature. The student uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length.

The student is expected to:

(A) estimate and measure length using nonstandard units such as paper clips or sides of color tiles;

(B) compare and order two or more concrete objects according to length (from longest to shortest);

(C) describe the relationship between the size of the unit and the number of units needed to measure the length of an object;

(D) compare and order the area of two or more two-dimensional surfaces (from covers the most to covers the least);

(E) compare and order two or more containers according to capacity (from holds the most to holds the least);

(F) compare and order two or more objects according to weight/mass (from heaviest to lightest); and

(G) compare and order two or more objects according to relative temperature (from hottest to coldest).

(1.8) **Measurement.** The student understands that time can be measured. The student uses time to describe and compare situations.

The student is expected to:

(A) order three or more events according to duration; and

(B) read time to the hour and half-hour using analog and digital clocks.

(1.9) **Probability and statistics.** The student displays data in an organized form.

The student is expected to:

(A) collect and sort data; and

(B) use organized data to construct real-object graphs, picture graphs, and bar-type graphs.

(1.10) **Probability and statistics.** The student uses information from organized data.

The student is expected to:

(A) draw conclusions and answer questions using information organized in real-object graphs, picture graphs, and bar-type graphs; and

(B) identify events as certain or impossible such as drawing a red crayon from a bag of green crayons.
(1.11) **Underlying processes and mathematical tools.** The student applies Grade 1 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify mathematics in everyday situations;

(B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and

(D) use tools such as real objects, manipulatives, and technology to solve problems.

(1.12) **Underlying processes and mathematical tools.** The student communicates about Grade 1 mathematics using informal language.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and

(B) relate informal language to mathematical language and symbols.

(1.13) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 2 are developing an understanding of the base-ten place value system, comparing and ordering whole numbers, applying addition and subtraction, and using measurement processes.

(2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.

(3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(2.1) Number, operation, and quantitative reasoning. The student understands how place value is used to represent whole numbers.

The student is expected to:

(A) use concrete models of hundreds, tens, and ones to represent a given whole number (up to 999) in various ways;

(B) use place value to read, write, and describe the value of whole numbers to 999; and

(C) use place value to compare and order whole numbers to 999 and record the comparisons using numbers and symbols (<, =, >).

(2.2) Number, operation, and quantitative reasoning. The student describes how fractions are used to name parts of whole objects or sets of objects.

The student is expected to:

(A) use concrete models to represent and name fractional parts of a whole object (with denominators of 12 or less); and

(B) use concrete models to represent and name fractional parts of a set of objects (with denominators of 12 or less); and

(C) use concrete models to determine if a fractional part of a whole is closer to 0, ½, or 1.
(2.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts whole numbers to solve problems.

The student is expected to:

(A) recall and apply basic addition and subtraction facts (to 18);

(B) model addition and subtraction of two-digit numbers with objects, pictures, words, and numbers;

(C) select addition or subtraction to solve problems using two-digit numbers, whether or not regrouping is necessary;

(D) determine the value of a collection of coins up to one dollar; and

(E) describe how the cent symbol, dollar symbol, and the decimal point are used to name the value of a collection of coins.

(2.4) **Number, operation, and quantitative reasoning.** The student models multiplication and division.

The student is expected to:

(A) model, create, and describe multiplication situations in which equivalent sets of concrete objects are joined; and

(B) model, create, and describe division situations in which a set of concrete objects is separated into equivalent sets.

(2.5) **Patterns, relationships, and algebraic thinking.** The student uses patterns in numbers and operations.

The student is expected to:

(A) find patterns in numbers such as in a 100s chart;

(B) use patterns in place value to compare and order whole numbers through 999; and

(C) use patterns and relationships to develop strategies to remember basic addition and subtraction facts. Determine patterns in related addition and subtraction number sentences (including fact families) such as 8 + 9 = 17, 9 + 8 = 17, 17 – 8 = 9, and 17 – 9 = 8.

(2.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to describe relationships and make predictions.

The student is expected to:

(A) generate a list of paired numbers based on a real-life situation such as number of tricycles related to number of wheels;

(B) identify patterns in a list of related number pairs based on a real-life situation and extend the list; and

(C) identify, describe, and extend repeating and additive patterns to make predictions and solve problems.
(2.7) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

The student is expected to:

(A) describe attributes (the number of vertices, faces, edges, sides) of two- and three-dimensional geometric figures such as circles, polygons, spheres, cones, cylinders, prisms, and pyramids, etc.;

(B) use attributes to describe how 2 two-dimensional figures or 2 three-dimensional geometric figures are alike or different; and

(C) cut two-dimensional geometric figures apart and identify the new geometric figures formed.

(2.8) **Geometry and spatial reasoning.** The student recognizes that a line can be used to represent a set of numbers and its properties.

The student is expected to use whole numbers to locate and name points on a number line.

(2.9) **Measurement.** The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length, area, capacity, and weight/mass. The student recognizes and uses models that approximate standard units (from both SI, also known as metric, and customary systems) of length, weight/mass, capacity, and time.

The student is expected to:

(A) identify concrete models that approximate standard units of length and use them to measure length;

(B) select a non-standard unit of measure such as square tiles to determine the area of a two-dimensional surface;

(C) select a non-standard unit of measure such as a bathroom cup or a jar to determine the capacity of a given container; and

(D) select a non-standard unit of measure such as beans or marbles to determine the weight/mass of a given object.

(2.10) **Measurement.** The student uses standard tools to estimate and measure time and temperature (in degrees Fahrenheit).

The student is expected to:

(A) read a thermometer to gather data;

(B) read and write times shown on analog and digital clocks using five-minute increments; and

(C) describe activities that take approximately one second, one minute, and one hour.

(2.11) **Probability and statistics.** The student organizes data to make it useful for interpreting information.

The student is expected to:

(A) construct picture graphs and bar-type graphs;

(B) draw conclusions and answer questions based on picture graphs and bar-type graphs; and

(C) use data to describe events as more likely or less likely such as drawing a certain color crayon from a bag of seven red crayons and three green crayons.
(2.12) **Underlying processes and mathematical tools.** The student applies Grade 2 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify the mathematics in everyday situations;
(B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
(D) use tools such as real objects, manipulatives, and technology to solve problems.

(2.13) **Underlying processes and mathematical tools.** The student communicates about Grade 2 mathematics using informal language.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and
(B) relate informal language to mathematical language and symbols.

(2.14) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.
§111.15. Mathematics, Grade 3.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 3 are multiplying and dividing whole numbers, connecting fraction symbols to fractional quantities, and standardizing language and procedures in geometry and measurement.

(2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.

(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(3.1) **Number, operation, and quantitative reasoning.** The student uses place value to communicate about increasingly large whole numbers in verbal and written form, including money.

The student is expected to:

(A) use place value to read, write (in symbols and words), and describe the value of whole numbers through 999,999;

(B) use place value to compare and order whole numbers through 9,999; and

(C) determine the value of a collection of coins and bills.

(3.2) **Number, operation, and quantitative reasoning.** The student uses fraction names and symbols (with denominators of 12 or less) to describe fractional parts of whole objects or sets of objects.

The student is expected to:

(A) construct concrete models of fractions;

(B) compare fractional parts of whole objects or sets of objects in a problem situation using concrete models;

(C) use fraction names and symbols to describe fractional parts of whole objects or sets of objects; and

(D) construct concrete models of equivalent fractions for fractional parts of whole objects.
(3.3) Number, operation, and quantitative reasoning. The student adds and subtracts to solve meaningful problems involving whole numbers.

The student is expected to:

(A) model addition and subtraction using pictures, words, and numbers; and

(B) select addition or subtraction and use the operation to solve problems involving whole numbers through 999.

(3.4) Number, operation, and quantitative reasoning. The student recognizes and solves problems in multiplication and division situations.

The student is expected to:

(A) learn and apply multiplication facts through 12 by 12 using concrete models and objects;

(B) solve and record multiplication problems (up to two digits times one digit); and

(C) use models to solve division problems and use number sentences to record the solutions.

(3.5) Number, operation, and quantitative reasoning. The student estimates to determine reasonable results.

The student is expected to:

(A) round whole numbers to the nearest ten or hundred to approximate reasonable results in problem situations; and

(B) use strategies including rounding and compatible numbers to estimate solutions to addition and subtraction problems.

(3.6) Patterns, relationships, and algebraic thinking. The student uses patterns to solve problems.

The student is expected to:

(A) identify and extend whole-number and geometric patterns to make predictions and solve problems;

(B) identify patterns in multiplication facts using concrete objects, pictorial models, or technology; and

(C) identify patterns in related multiplication and division sentences (fact families) such as $2 \times 3 = 6$, $3 \times 2 = 6$, $6 \div 2 = 3$, $6 \div 3 = 2$.

(3.7) Patterns, relationships, and algebraic thinking. The student uses lists, tables, and charts to express patterns and relationships.

The student is expected to:

(A) generate a table of paired numbers based on a real-life situation such as insects and legs; and

(B) identify and describe patterns in a table of related number pairs based on a meaningful problem and extend the table.
(3.8) **Geometry and spatial reasoning.** The student uses formal geometric vocabulary.

The student is expected to identify, classify, and describe two- and three-dimensional geometric figures by their attributes. The student compares two-dimensional figures, three-dimensional figures, or both by their attributes using formal geometry vocabulary.

(3.9) **Geometry and spatial reasoning.** The student recognizes congruence and symmetry.

The student is expected to:

(A) identify congruent two-dimensional figures;

(B) create two-dimensional figures with lines of symmetry using concrete models and technology; and

(C) identify lines of symmetry in two-dimensional geometric figures.

(3.10) **Geometry and spatial reasoning.** The student recognizes that a line can be used to represent numbers and fractions and their properties and relationships.

The student is expected to locate and name points on a number line using whole numbers and fractions, including halves and fourths.

(3.11) **Measurement.** The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses standard units to describe length, area, capacity/volume, and weight/mass.

The student is expected to:

(A) use linear measurement tools to estimate and measure lengths using standard units;

(B) use standard units to find the perimeter of a shape;

(C) use concrete and pictorial models of square units to determine the area of two-dimensional surfaces;

(D) identify concrete models that approximate standard units of weight/mass and use them to measure weight/mass;

(E) identify concrete models that approximate standard units for capacity and use them to measure capacity; and

(F) use concrete models that approximate cubic units to determine the volume of a given container or other three-dimensional geometric figure.

(3.12) **Measurement.** The student reads and writes time and measures temperature in degrees Fahrenheit to solve problems.

The student is expected to:

(A) use a thermometer to measure temperature; and

(B) tell and write time shown on analog and digital clocks.
(3.13) **Probability and statistics.**
The student solves problems by collecting, organizing, displaying, and interpreting sets of data.

The student is expected to:
(A) collect, organize, record, and display data in pictographs and bar graphs where each picture or cell might represent more than one piece of data;
(B) interpret information from pictographs and bar graphs; and
(C) use data to describe events as more likely than, less likely than, or equally likely as.

(3.14) **Underlying processes and mathematical tools.** The student applies Grade 3 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:
(A) identify the mathematics in everyday situations;
(B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
(D) use tools such as real objects, manipulatives, and technology to solve problems.

(3.15) **Underlying processes and mathematical tools.** The student communicates about Grade 3 mathematics using informal language.

The student is expected to:
(A) explain and record observations using objects, words, pictures, numbers, and technology; and
(B) relate informal language to mathematical language and symbols.

(3.16) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to:
(A) make generalizations from patterns or sets of examples and nonexamples; and
(B) justify why an answer is reasonable and explain the solution process.
§111.16. Mathematics, Grade 4.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 4 are comparing and ordering fractions and decimals, applying multiplication and division, and developing ideas related to congruence and symmetry.

(2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.

(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

The student is expected to:

(4.1) **Number, operation, and quantitative reasoning.** The student uses place value to represent whole numbers and decimals.

(A) use place value to read, write, compare, and order whole numbers through 999,999,999; and

(B) use place value to read, write, compare, and order decimals involving tenths and hundredths, including money, using concrete objects and pictorial models.

(4.2) **Number, operation, and quantitative reasoning.** The student describes and compares fractional parts of whole objects or sets of objects.

(A) use concrete objects and pictorial models to generate equivalent fractions;

(B) model fraction quantities greater than one using concrete objects and pictorial models;

(C) compare and order fractions using concrete objects and pictorial models; and

(D) relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models.
(4.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts to solve meaningful problems involving whole numbers and decimals.

The student is expected to:

(A) use addition and subtraction to solve problems involving whole numbers; and

(B) add and subtract decimals to the hundredths place using concrete objects and pictorial models.

(4.4) **Number, operation, and quantitative reasoning.** The student multiplies and divides to solve meaningful problems involving whole numbers.

The student is expected to:

(A) model factors and products using arrays and area models;

(B) represent multiplication and division situations in picture, word, and number form;

(C) recall and apply multiplication facts through $12 \times 12$;

(D) use multiplication to solve problems (no more than two digits times two digits without technology); and

(E) use division to solve problems (no more than one-digit divisors and three-digit dividends without technology).

(4.5) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.

The student is expected to:

(A) round whole numbers to the nearest ten, hundred, or thousand to approximate reasonable results in problem situations; and

(B) use strategies including rounding and compatible numbers to estimate solutions to multiplication and division problems.

(4.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns in multiplication and division.

The student is expected to:

(A) use patterns and relationships to develop strategies to remember basic multiplication and division facts (such as the patterns in related multiplication and division number sentences (fact families) such as $9 \times 9 = 81$ and $81 \div 9 = 9$); and

(B) use patterns to multiply by 10 and 100.

(4.7) **Patterns, relationships, and algebraic thinking.** The student uses organizational structures to analyze and describe patterns and relationships.

The student is expected to describe the relationship between two sets of related data such as ordered pairs in a table.
(4.8) **Geometry and spatial reasoning.** The student identifies and describes attributes of geometric figures using formal geometric language.

The student is expected to:

(A) identify and describe right, acute, and obtuse angles;
(B) identify and describe parallel and intersecting (including perpendicular) lines using concrete objects and pictorial models; and
(C) use essential attributes to define two- and three-dimensional geometric figures.

(4.9) **Geometry and spatial reasoning.** The student connects transformations to congruence and symmetry.

The student is expected to:

(A) demonstrate translations, reflections, and rotations using concrete models;
(B) use translations, reflections, and rotations to verify that two shapes are congruent; and
(C) use reflections to verify that a shape has symmetry.

The student is expected to locate and name points on a number line using whole numbers, fractions such as halves and fourths, and decimals such as tenths.

(4.10) **Geometry and spatial reasoning.** The student recognizes the connection between numbers and their properties and points on a line.

(4.11) **Measurement.** The student applies measurement concepts. The student is expected to estimate and measure to solve problems involving length (including perimeter) and area. The student uses measurement tools to measure capacity/volume and weight/mass.

The student is expected to:

(A) estimate and use measurement tools to determine length (including perimeter), area, capacity and weight/mass using standard units SI (metric) and customary;
(B) perform simple conversions between different units of length, between different units of capacity, and between different units of weight within the customary measurement system;
(C) use concrete models of standard cubic units to measure volume;
(D) estimate volume in cubic units; and
(E) explain the difference between weight and mass.

(4.12) **Measurement.** The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius).

The student is expected to:

(A) use a thermometer to measure temperature and changes in temperature; and
(B) use tools such as a clock with gears or a stopwatch to solve problems involving elapsed time.
(4.13) **Probability and statistics.**
The student solves problems by collecting, organizing, displaying, and interpreting sets of data.

The student is expected to:

(A) use concrete objects or pictures to make generalizations about determining all possible combinations of a given set of data or of objects in a problem situation; and

(B) interpret bar graphs.

(4.14) **Underlying processes and mathematical tools.** The student applies Grade 4 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify the mathematics in everyday situations;

(B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) use tools such as real objects, manipulatives, and technology to solve problems.

(4.15) **Underlying processes and mathematical tools.** The student communicates about Grade 4 mathematics using informal language.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and

(B) relate informal language to mathematical language and symbols.

(4.16) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to:

(A) make generalizations from patterns or sets of examples and nonexamples; and

(B) justify why an answer is reasonable and explain the solution process.
§111.17. Mathematics, Grade 5.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 5 are comparing and contrasting lengths, areas, and volumes of two- or three-dimensional geometric figures; representing and interpreting data in graphs, charts, and tables; and applying whole number operations in a variety of contexts.

(2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.

(3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.

(4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

(b) Knowledge and skills.

(5.1) **Number, operation, and quantitative reasoning.** The student uses place value to represent whole numbers and decimals.

The student is expected to:

(A) use place value to read, write, compare, and order whole numbers through the 999,999,999,999; and

(B) use place value to read, write, compare, and order decimals through the thousandths place.

(5.2) **Number, operation, and quantitative reasoning.** The student uses fractions in problem-solving situations.

The student is expected to:

(A) generate a fraction equivalent to a given fraction such as 1/2 and 3/6 or 4/12 and 1/3;

(B) generate a mixed number equivalent to a given improper fraction or generate an improper fraction equivalent to a given mixed number;

(C) compare two fractional quantities in problem-solving situations using a variety of methods, including common denominators; and

(D) use models to relate decimals to fractions that name tenths, hundredths, and thousandths.
(5.3) **Number, operation, and quantitative reasoning.** The student adds, subtracts, multiplies, and divides to solve meaningful problems.

The student is expected to:

(A) use addition and subtraction to solve problems involving whole numbers and decimals;

(B) use multiplication to solve problems involving whole numbers (no more than three digits times two digits without technology);

(C) use division to solve problems involving whole numbers (no more than two-digit divisors and three-digit dividends without technology), including interpreting the remainder within a given context;

(D) identify common factors of a set of whole numbers; and

(E) model situations using addition and/or subtraction involving fractions with like denominators using concrete objects, pictures, words, and numbers.

(5.4) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.

The student is expected to use strategies, including rounding and compatible numbers to estimate solutions to addition, subtraction, multiplication, and division problems.

(5.5) **Patterns, relationships, and algebraic thinking.** The student makes generalizations based on observed patterns and relationships.

The student is expected to:

(A) describe the relationship between sets of data in graphic organizers such as lists, tables, charts, and diagrams; and

(B) identify prime and composite numbers using concrete objects, pictorial models, and patterns in factor pairs.

(5.6) **Patterns, relationships, and algebraic thinking.** The student describes relationships mathematically.

The student is expected to select from and use diagrams and equations such as $y = 5 + 3$ to represent meaningful problem situations.

(5.7) **Geometry and spatial reasoning.** The student generates geometric definitions using critical attributes.

The student is expected to identify essential attributes including parallel, perpendicular, and congruent parts of two- and three-dimensional geometric figures.

(5.8) **Geometry and spatial reasoning.** The student models transformations.

The student is expected to:

(A) sketch the results of translations, rotations, and reflections on a Quadrant I coordinate grid; and

(B) identify the transformation that generates one figure from the other when given two congruent figures on a Quadrant I coordinate grid.
(5.9) **Geometry and spatial reasoning.** The student recognizes the connection between ordered pairs of numbers and locations of points on a plane.

The student is expected to locate and name points on a coordinate grid using ordered pairs of whole numbers.

(5.10) **Measurement.** The student applies measurement concepts involving length (including perimeter), area, capacity/volume, and weight/mass to solve problems.

The student is expected to:

(A) perform simple conversions within the same measurement system (SI (metric) or customary);

(B) connect models for perimeter, area, and volume with their respective formulas; and

(C) select and use appropriate units and formulas to measure length, perimeter, area, and volume.

(5.11) **Measurement.** The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius).

The student is expected to:

(A) solve problems involving changes in temperature; and

(B) solve problems involving elapsed time.

(5.12) **Probability and statistics.** The student describes and predicts the results of a probability experiment.

The student is expected to:

(A) use fractions to describe the results of an experiment;

(B) use experimental results to make predictions; and

(C) list all possible outcomes of a probability experiment such as tossing a coin.

(5.13) **Probability and statistics.** The student solves problems by collecting, organizing, displaying, and interpreting sets of data.

The student is expected to:

(A) use tables of related number pairs to make line graphs;

(B) describe characteristics of data presented in tables and graphs including median, mode, and range; and

(C) graph a given set of data using an appropriate graphical representation such as a picture or line graph.
(5.14) Underlying processes and mathematical tools. The student applies Grade 5 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

(A) identify the mathematics in everyday situations;

(B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) use tools such as real objects, manipulatives, and technology to solve problems.

(5.15) Underlying processes and mathematical tools. The student communicates about Grade 5 mathematics using informal language.

The student is expected to:

(A) explain and record observations using objects, words, pictures, numbers, and technology; and

(B) relate informal language to mathematical language and symbols.

(5.16) Underlying processes and mathematical tools. The student uses logical reasoning.

The student is expected to:

(A) make generalizations from patterns or sets of examples and nonexamples; and

(B) justify why an answer is reasonable and explain the solution process.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter B. Middle School
Text of Adopted Amendments to 19 TAC

§111.21. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 6-8.

The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year.

§111.22. Mathematics, Grade 6.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 6 are using ratios to describe direct proportional relationships involving number, geometry, measurement, probability, and adding and subtracting decimals and fractions.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

(6.1) **Number, operation, and quantitative reasoning.** The student represents and uses rational numbers in a variety of equivalent forms.

The student is expected to:

(A) compare and order non-negative rational numbers;

(B) generate equivalent forms of rational numbers including whole numbers, fractions, and decimals;

(C) use integers to represent real-life situations;

(D) write prime factorizations using exponents;

(E) identify factors of a positive integer, common factors, and the greatest common factor of a set of positive integers; and

(F) identify multiples of a positive integer and common multiples and the least common multiple of a set of positive integers.
(6.2) **Number, operation, and quantitative reasoning.**
The student adds, subtracts, multiplies, and divides to solve problems and justify solutions.

The student is expected to:
(A) model addition and subtraction situations involving fractions with objects, pictures, words, and numbers;
(B) use addition and subtraction to solve problems involving fractions and decimals;
(C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates;
(D) estimate and round to approximate reasonable results and to solve problems where exact answers are not required; and
(E) use order of operations to simplify whole number expressions (without exponents) in problem solving situations.

(6.3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving direct proportional relationships.

The student is expected to:
(A) use ratios to describe proportional situations;
(B) represent ratios and percents with concrete models, fractions, and decimals; and
(C) use ratios to make predictions in proportional situations.

(6.4) **Patterns, relationships, and algebraic thinking.** The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes.

The student is expected to:
(A) use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter and area; and
(B) use tables of data to generate formulas representing relationships involving perimeter, area, volume of a rectangular prism, etc.

(6.5) **Patterns, relationships, and algebraic thinking.** The student uses letters to represent an unknown in an equation.

The student is expected to formulate equations from problem situations described by linear relationships.

(6.6) **Geometry and spatial reasoning.** The student uses geometric vocabulary to describe angles, polygons, and circles.

The student is expected to:
(A) use angle measurements to classify angles as acute, obtuse, or right;
(B) identify relationships involving angles in triangles and quadrilaterals; and
(C) describe the relationship between radius, diameter, and circumference of a circle.
(6.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to identify location in two dimensions.

The student is expected to locate and name points on a coordinate plane using ordered pairs of non-negative rational numbers.

(6.8) **Measurement.** The student solves application problems involving estimation and measurement of length, area, time, temperature, volume, weight, and angles.

The student is expected to:

(A) estimate measurements (including circumference) and evaluate reasonableness of results;

(B) select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter), area, time, temperature, volume, and weight;

(C) measure angles; and

(D) convert measures within the same measurement system (customary and metric) based on relationships between units.

(6.9) **Probability and statistics.** The student uses experimental and theoretical probability to make predictions.

The student is expected to:

(A) construct sample spaces using lists and tree diagrams; and

(B) find the probabilities of a simple event and its complement and describe the relationship between the two.

(6.10) **Probability and statistics.** The student uses statistical representations to analyze data.

The student is expected to:

(A) select and use an appropriate representation for presenting and displaying different graphical representations of the same data including line plot, line graph, bar graph, and stem and leaf plot;

(B) identify mean (using concrete objects and pictorial models), median, mode, and range of a set of data;

(C) sketch circle graphs to display data; and

(D) solve problems by collecting, organizing, displaying, and interpreting data.
(6.11) **Underlying processes and mathematical tools.** The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

(6.12) **Underlying processes and mathematical tools.** The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models.

The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and

(B) evaluate the effectiveness of different representations to communicate ideas.

(6.13) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:

(A) make conjectures from patterns or sets of examples and nonexamples; and

(B) validate his/her conclusions using mathematical properties and relationships.
§111.23. Mathematics, Grade 7.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 7 are using direct proportional relationships in number, geometry, measurement, and probability; applying addition, subtraction, multiplication, and division of decimals, fractions, and integers; and using statistical measures to describe data.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

(7.1) **Number, operation, and quantitative reasoning.** The student represents and uses numbers in a variety of equivalent forms.

The student is expected to:

(A) compare and order integers and positive rational numbers;

(B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator; and

(C) represent squares and square roots using geometric models.
(7.2) **Number, operation, and quantitative reasoning.**

The student adds, subtracts, multiplies, or divides to solve problems and justify solutions.

The student is expected to:

(A) represent multiplication and division situations involving fractions and decimals with models, including concrete objects, pictures, words, and numbers;

(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals;

(C) use models, such as concrete objects, pictorial models, and number lines, to add, subtract, multiply, and divide integers and connect the actions to algorithms;

(D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio;

(E) simplify numerical expressions involving order of operations and exponents;

(F) select and use appropriate operations to solve problems and justify the selections; and

(G) determine the reasonableness of a solution to a problem.

(7.3) **Patterns, relationships, and algebraic thinking.**

The student solves problems involving direct proportional relationships.

The student is expected to:

(A) estimate and find solutions to application problems involving percent; and

(B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

(7.4) **Patterns, relationships, and algebraic thinking.**

The student represents a relationship in numerical, geometric, verbal, and symbolic form.

The student is expected to:

(A) generate formulas involving unit conversions within the same system (customary and metric), perimeter, area, circumference, volume, and scaling;

(B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling; and

(C) use words and symbols to describe the relationship between the terms in an arithmetic sequence (with a constant rate of change) and their positions in the sequence.
(7.5) **Patterns, relationships, and algebraic thinking.** The student uses equations to solve problems.

(7.6) **Geometry and spatial reasoning.** The student compares and classifies two- and three-dimensional figures using geometric vocabulary and properties.

(7.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to describe location on a plane.

(7.8) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world.

(7.9) **Measurement.** The student solves application problems involving estimation and measurement.

The student is expected to:

(A) use concrete and pictorial models to solve equations and use symbols to record the actions; and

(B) formulate problem situations when given a simple equation and formulate an equation when given a problem situation.

The student is expected to:

(A) use angle measurements to classify pairs of angles as complementary or supplementary;

(B) use properties to classify triangles and quadrilaterals;

(C) use properties to classify three-dimensional figures, including pyramids, cones, prisms, and cylinders; and

(D) use critical attributes to define similarity.

The student is expected to:

(A) locate and name points on a coordinate plane using ordered pairs of integers; and

(B) graph reflections across the horizontal or vertical axis and graph translations on a coordinate plane.

The student is expected to:

(A) sketch three-dimensional figures when given the top, side, and front views;

(B) make a net (two-dimensional model) of the surface area of a three-dimensional figure; and

(C) use geometric concepts and properties to solve problems in fields such as art and architecture.

The student is expected to:

(A) estimate measurements and solve application problems involving length (including perimeter and circumference) and area of polygons and other shapes;

(B) connect models for volume of prisms (triangular and rectangular) and cylinders to formulas of prisms (triangular and rectangular) and cylinders; and

(C) estimate measurements and solve application problems involving volume of prisms (rectangular and triangular) and cylinders.
(7.10) **Probability and statistics.** The student recognizes that a physical or mathematical model (including geometric) can be used to describe the experimental and theoretical probability of real-life events.

The student is expected to:

(A) construct sample spaces for simple or composite experiments; and

(B) find the probability of independent events.

(7.11) **Probability and statistics.** The student understands that the way a set of data is displayed influences its interpretation.

The student is expected to:

(A) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection; and

(B) make inferences and convincing arguments based on an analysis of given or collected data.

(7.12) **Probability and statistics.** The student uses measures of central tendency and variability to describe a set of data.

The student is expected to:

(A) describe a set of data using mean, median, mode, and range; and

(B) choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation.

(7.13) **Underlying processes and mathematical tools.** The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

The student is expected to:

(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
(7.14) **Underlying processes and mathematical tools.** The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models.

The student is expected to:

- (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
- (B) evaluate the effectiveness of different representations to communicate ideas.

(7.15) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:

- (A) make conjectures from patterns or sets of examples and nonexamples; and
- (B) validate his/her conclusions using mathematical properties and relationships.
§111.24. Mathematics, Grade 8.

(a) Introduction.

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 8 are using basic principles of algebra to analyze and represent both proportional and non-proportional linear relationships and using probability to describe data and make predictions.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.

(3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

(8.1) **Number, operation, and quantitative reasoning.** The student understands that different forms of numbers are appropriate for different situations.

The student is expected to:

(A) compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals;

(B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships;

(C) approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (such as \(\pi\), \(\sqrt{2}\));

(D) express numbers in scientific notation, including negative exponents, in appropriate problem situations; and

(E) compare and order real numbers with a calculator.

(8.2) **Number, operation, and quantitative reasoning.** The student selects and uses appropriate operations to solve problems and justify solutions.

The student is expected to:

(A) select appropriate operations to solve problems involving rational numbers and justify the selections;

(B) use appropriate operations to solve problems involving rational numbers in problem situations;

(C) evaluate a solution for reasonableness; and
(8.3) **Patterns, relationships, and algebraic thinking.** The student identifies proportional or non-proportional linear relationships in problem situations and solves problems.

The student is expected to:

(A) compare and contrast proportional and non-proportional linear relationships; and

(B) estimate and find solutions to application problems involving percents and other proportional relationships such as similarity and rates.

(8.4) **Patterns, relationships, and algebraic thinking.** The student makes connections among various representations of a numerical relationship.

The student is expected to generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description).

(8.5) **Patterns, relationships, and algebraic thinking.** The student uses graphs, tables, and algebraic representations to make predictions and solve problems.

The student is expected to:

(A) predict, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations; and

(B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).

(8.6) **Geometry and spatial reasoning.** The student uses transformational geometry to develop spatial sense.

The student is expected to:

(A) generate similar figures using dilations including enlargements and reductions; and

(B) graph dilations, reflections, and translations on a coordinate plane.

(8.7) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world.

The student is expected to:

(A) draw three-dimensional figures from different perspectives;

(B) use geometric concepts and properties to solve problems in fields such as art and architecture;

(C) use pictures or models to demonstrate the Pythagorean Theorem; and

(D) locate and name points on a coordinate plane using ordered pairs of rational numbers.
(8.8) **Measurement.** The student uses procedures to determine measures of three-dimensional figures.

The student is expected to:

(A) find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models);

(B) connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects; and

(C) estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.

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(8.9) **Measurement.** The student uses indirect measurement to solve problems.

The student is expected to:

(A) use the Pythagorean Theorem to solve real-life problems; and

(B) use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements.

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(8.10) **Measurement.** The student describes how changes in dimensions affect linear, area, and volume measures.

The student is expected to:

(A) describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally; and

(B) describe the resulting effect on volume when dimensions of a solid are changed proportionally.

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(8.11) **Probability and statistics.** The student applies concepts of theoretical and experimental probability to make predictions.

The student is expected to:

(A) find the probabilities of dependent and independent events;

(B) use theoretical probabilities and experimental results to make predictions and decisions; and

(C) select and use different models to simulate an event.
(8.12) **Probability and statistics.** The student uses statistical procedures to describe data. The student is expected to:

- (A) use variability (range, including interquartile range (IQR)) and select the appropriate measure of central tendency to describe a set of data and justify the choice for a particular situation;
- (B) draw conclusions and make predictions by analyzing trends in scatterplots; and
- (C) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, bar graphs, box and whisker plots, histograms, and Venn diagrams, with and without the use of technology.

(8.13) **Probability and statistics.** The student evaluates predictions and conclusions based on statistical data. The student is expected to:

- (A) evaluate methods of sampling to determine validity of an inference made from a set of data; and
- (B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.

(8.14) **Underlying processes and mathematical tools.** The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

- (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
- (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
- (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
- (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
(8.15) **Underlying processes and mathematical tools.** The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models.

The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and

(B) evaluate the effectiveness of different representations to communicate ideas.

(8.16) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.

The student is expected to:

(A) make conjectures from patterns or sets of examples and nonexamples; and

(B) validate his/her conclusions using mathematical properties and relationships.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter C. High School
Text of Adopted Amendments to 19 TAC

§111.31. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 9-12.

The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year.

§111.32. Algebra I (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students will continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students use symbols in a variety of ways to study relationships among quantities.

(3) Function concepts. A function is a fundamental mathematical concept; it expresses a special kind of relationship between two quantities. Students use functions to determine one quantity from another, to represent and model problem situations, and to analyze and interpret relationships.

(4) Relationship between equations and functions. Equations and inequalities arise as a way of asking and answering questions involving functional relationships. Students work in many situations to set up equations and inequalities and use a variety of methods to solve them.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(b) Knowledge and skills.

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

(B) gather and record data and use data sets to determine functional relationships between quantities;

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;
(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

The student is expected to:

(A) identify and sketch the general forms of linear \((y = x)\) and quadratic \((y = x^2)\) parent functions;

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and

(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and

(C) connect equation notation with function notation, such as \(y = x + 1\) and \(f(x) = x + 1\).

The student is expected to:

(A) determine whether or not given situations can be represented by linear functions;

(B) determine the domain and range for linear functions in given situations; and

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.
(A.6) **Linear functions.** The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigate, describe, and predict the effects of changes in \( m \) and \( b \) on the graph of \( y = mx + b \);

(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and \( y \)-intercept;

(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations;

(F) interpret and predict the effects of changing slope and \( y \)-intercept in applied situations; and

(G) relate direct variation to linear functions and solve problems involving proportional change.

(A.7) **Linear functions.** The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

(C) interpret and determine the reasonableness of solutions to linear equations and inequalities.

(A.8) **Linear functions.** The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) interpret and determine the reasonableness of solutions to systems of linear equations.
(A.9) **Quadratic and other nonlinear functions.** The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

(B) investigate, describe, and predict the effects of changes in \( a \) on the graph of \( y = ax^2 + c \);

(C) investigate, describe, and predict the effects of changes in \( c \) on the graph of \( y = ax^2 + c \); and

(D) analyze graphs of quadratic functions and draw conclusions.

(A.10) **Quadratic and other nonlinear functions.** The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

(A.11) **Quadratic and other nonlinear functions.** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:

(A) use patterns to generate the laws of exponents and apply them in problem-solving situations;

(B) analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods; and

(C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.
§111.33. Mathematics, Algebra II

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students study algebraic concepts and the relationships among them to better understand the structure of algebra.

(3) Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics. Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.

(4) Relationship between algebra and geometry. Equations and functions are algebraic tools that can be used to represent geometric curves and figures; similarly, geometric figures can illustrate algebraic relationships. Students perceive the connections between algebra and geometry and use the tools of one to help solve problems in the other.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(b) Knowledge and skills.

(2A.1) Foundations for functions.

The student uses properties and attributes of functions and applies functions to problem situations.

The student is expected to:

(A) identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations; and

(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

(2A.2) Foundations for functions.

The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations; and

(B) use complex numbers to describe the solutions of quadratic equations.
(2A.3) **Foundations for functions.** The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations.

The student is expected to:

(A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems;

(B) use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities; and

(C) interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.

(2A.4) **Algebra and geometry.** The student connects algebraic and geometric representations of functions.

The student is expected to:

(A) identify and sketch graphs of parent functions, including linear \((f(x) = x)\), quadratic \((f(x) = x^2)\), exponential \((f(x) = a^x)\), and logarithmic \((f(x) = \log a x)\) functions, absolute value of \(x\) \((f(x) = |x|)\), square root of \(x\) \((f(x) = \sqrt{x})\), and reciprocal of \(x\) \((f(x) = 1/x)\);

(B) extend parent functions with parameters such as \(a\) in \(f(x) = a/x\) and describe the effects of the parameter changes on the graph of parent functions; and

(C) describe and analyze the relationship between a function and its inverse.

(2A.5) **Algebra and geometry.** The student knows the relationship between the geometric and algebraic descriptions of conic sections.

The student is expected to:

(A) describe a conic section as the intersection of a plane and a cone;

(B) sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph;

(C) identify symmetries from graphs of conic sections;

(D) identify the conic section from a given equation; and

(E) use the method of completing the square.

(2A.6) **Quadratic and square root functions.** The student understands that quadratic functions can be represented in different ways and translates among their various representations.

The student is expected to:

(A) determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities;

(B) relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions; and

(C) determine a quadratic function from its roots (real and complex) or a graph.
(2A.7) **Quadratic and square root functions.** The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.

The student is expected to:

(A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the \( y = ax^2 + bx + c \) and the \( y = a(x - h)^2 + k \) symbolic representations of quadratic functions; and

(B) use the parent function to investigate, describe, and predict the effects of changes in \( a \), \( h \), and \( k \) on the graphs of \( y = a(x - h)^2 + k \) form of a function in applied and purely mathematical situations.

(2A.8) **Quadratic and square root functions.** The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems;

(B) analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula;

(C) compare and translate between algebraic and graphical solutions of quadratic equations; and

(D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.

(2A.9) **Quadratic and square root functions.** The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges;

(B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions;

(C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities;

(D) determine solutions of square root equations using graphs, tables, and algebraic methods;

(E) determine solutions of square root inequalities using graphs and tables;

(F) analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems; and

(G) connect inverses of square root functions with quadratic functions.
(2A.10) **Rational functions.** The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior;

(B) analyze various representations of rational functions with respect to problem situations;

(C) determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities;

(D) determine the solutions of rational equations using graphs, tables, and algebraic methods;

(E) determine solutions of rational inequalities using graphs and tables;

(F) analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and

(G) use functions to model and make predictions in problem situations involving direct and inverse variation.

(2A.11) **Exponential and logarithmic functions.** The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses;

(B) use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior;

(C) determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities;

(D) determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods;

(E) determine solutions of exponential and logarithmic inequalities using graphs and tables; and

(F) analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.
§111.34. Geometry (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; geometric figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.

(3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.

(4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

(b) Knowledge and skills.

(G.1) Geometric structure. The student understands the structure of, and relationships within, an axiomatic system.

The student is expected to:

(A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;

(B) recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes; and

(C) compare and contrast the structures and implications of Euclidean and non-Euclidean geometries.
(G.2) **Geometric structure.** The student analyzes geometric relationships in order to make and verify conjectures.

The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and

(B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(G.3) **Geometric structure.** The student applies logical reasoning to justify and prove mathematical statements.

The student is expected to:

(A) determine the validity of a conditional statement, its converse, inverse, and contrapositive;

(B) construct and justify statements about geometric figures and their properties;

(C) use logical reasoning to prove statements are true and find counter examples to disprove statements that are false;

(D) use inductive reasoning to formulate a conjecture; and

(E) use deductive reasoning to prove a statement.

(G.4) **Geometric structure.** The student uses a variety of representations to describe geometric relationships and solve problems.

The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

(G.5) **Geometric patterns.** The student uses a variety of representations to describe geometric relationships and solve problems.

The student is expected to:

(A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties;

(B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;

(C) use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations; and

(D) identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.
(G.6) **Dimensionality and the geometry of location.** The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems.

The student is expected to:

(A) describe and draw the intersection of a given plane with various three-dimensional geometric figures;

(B) use nets to represent and construct three-dimensional geometric figures; and

(C) use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems.

(G.7) **Dimensionality and the geometry of location.** The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.

The student is expected to:

(A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;

(B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and

(C) derive and use formulas involving length, slope, and midpoint.

(G.8) **Congruence and the geometry of size.** The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.

The student is expected to:

(A) find areas of regular polygons, circles, and composite figures;

(B) find areas of sectors and arc lengths of circles using proportional reasoning;

(C) derive, extend, and use the Pythagorean Theorem;

(D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations;

(E) use area models to connect geometry to probability and statistics; and

(F) use conversions between measurement systems to solve problems in real-world situations.
(G.9) **Congruence and the geometry of size.** The student analyzes properties and describes relationships in geometric figures.

The student is expected to:

(A) formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models;

(B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;

(C) formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models; and

(D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.

(G.10) **Congruence and the geometry of size.** The student applies the concept of congruence to justify properties of figures and solve problems.

The student is expected to:

(A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and

(B) justify and apply triangle congruence relationships.

(G.11) **Similarity and the geometry of shape.** The student applies the concepts of similarity to justify properties of figures and solve problems.

The student is expected to:

(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures;

(B) use ratios to solve problems involving similar figures;

(C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and

(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.
§111.35. Precalculus (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998, and at that time shall supersede §75.63(bb) of this title (relating to Mathematics). Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.

(b) Introduction.

(1) In Precalculus, students continue to build on the K-8, Algebra I, Algebra II, and Geometry foundations as they expand their understanding through other mathematical experiences. Students use symbolic reasoning and analytical methods to represent mathematical situations, to express generalizations, and to study mathematical concepts and the relationships among them. Students use functions, equations, and limits as useful tools for expressing generalizations and as means for analyzing and understanding a broad variety of mathematical relationships. Students also use functions as well as symbolic reasoning to represent and connect ideas in geometry, probability, statistics, trigonometry, and calculus and to model physical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model functions and equations and solve real-life problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(c) Knowledge and skills.

(P.1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions.

The student is expected to:

(A) describe parent functions symbolically and graphically, including \( f(x) = x^n, f(x) = \ln x, f(x) = \log_a x, f(x) = \frac{1}{x}, f(x) = e^x, f(x) = |x|, f(x) = a^x, f(x) = \sin x, f(x) = \arcsin x, \) etc.;

(B) determine the domain and range of functions using graphs, tables, and symbols;

(C) describe symmetry of graphs of even and odd functions;

(D) recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc.), points on the graph of a function, and the symbolic representation of a function; and

(E) investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically.
(P.2) The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems.

The student is expected to:

(A) apply basic transformations, including \( a \cdot f(x) \), \( f(x) + d \), \( f(x - c) \), \( f(b \cdot x) \), and compositions with absolute value functions, including \(|f(x)|\), and \( f(|x|) \), to the parent functions;

(B) perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically; and

(C) investigate identities graphically and verify them symbolically, including logarithmic properties, trigonometric identities, and exponential properties.

(P.3) The student uses functions and their properties, tools and technology, to model and solve meaningful problems.

The student is expected to:

(A) investigate properties of trigonometric and polynomial functions;

(B) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data;

(C) use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient);

(D) use properties of functions to analyze and solve problems and make predictions; and

(E) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed.

(P.4) The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems.

The student is expected to:

(A) represent patterns using arithmetic and geometric sequences and series;

(B) use arithmetic, geometric, and other sequences and series to solve real-life problems;

(C) describe limits of sequences and apply their properties to investigate convergent and divergent series; and

(D) apply sequences and series to solve problems including sums and binomial expansion.
(P.5) The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations.

The student is expected to:

(A) use conic sections to model motion, such as the graph of velocity vs. position of a pendulum and motions of planets;

(B) use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound;

(C) convert between parametric and rectangular forms of functions and equations to graph them; and

(D) use parametric functions to simulate problems involving motion.

(P.6) The student uses vectors to model physical situations.

The student is expected to:

(A) use the concept of vectors to model situations defined by magnitude and direction; and

(B) analyze and solve vector problems generated by real-life situations.
§111.36. Mathematical Models with Applications (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Algebra I.

(b) Introduction.

(1) In Mathematical Models with Applications, students continue to build on the K-8 and Algebra I foundations as they expand their understanding through other mathematical experiences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, to model information, and to solve problems from various disciplines. Students use mathematical methods to model and solve real-life applied problems involving money, data, chance, patterns, music, design, and science. Students use mathematical models from algebra, geometry, probability, and statistics and connections among these to solve problems from a wide variety of advanced applications in both mathematical and nonmathematical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to link modeling techniques and purely mathematical concepts and to solve applied problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(c) Knowledge and skills.

(M.1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems.

The student is expected to:

(A) compare and analyze various methods for solving a real-life problem;

(B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and

(C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.

(M.2) The student uses graphical and numerical techniques to study patterns and analyze data.

The student is expected to:

(A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, line plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data;

(B) analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences;

(C) analyze graphs from journals, newspapers, and other sources to determine the validity of stated arguments; and

(D) use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information.
§111.36. Mathematics, Mathematical Models with Applications

(M.3) The student develops and implements a plan for collecting and analyzing data (qualitative and quantitative) in order to make decisions.

The student is expected to:
(A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions;
(B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project by written report, visual display, oral report, or multi-media presentation; and
(C) determine the appropriateness of a model for making predictions from a given set of data.

(M.4) The student uses probability models to describe everyday situations involving chance.

The student is expected to:
(A) compare theoretical and empirical probability; and
(B) use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc.

(M.5) The student uses functional relationships to solve problems related to personal income.

The student is expected to:
(A) use rates, linear functions, and direct variation to solve problems involving personal finance and budgeting, including compensations and deductions;
(B) solve problems involving personal taxes; and
(C) analyze data to make decisions about banking.

(M.6) The student uses algebraic formulas, graphs, and amortization models to solve problems involving credit.

The student is expected to:
(A) analyze methods of payment available in retail purchasing and compare relative advantages and disadvantages of each option;
(B) use amortization models to investigate home financing and compare buying and renting a home; and
(C) use amortization models to investigate automobile financing and compare buying and leasing a vehicle.

(M.7) The student uses algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning.

The student is expected to:
(A) analyze types of savings options involving simple and compound interest and compare relative advantages of these options;
(B) analyze and compare coverage options and rates in insurance; and
(C) investigate and compare investment options including stocks, bonds, annuities, and retirement plans.
(M.8) The student uses algebraic and geometric models to describe situations and solve problems.

The student is expected to:

(A) use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology;

(B) use trigonometric ratios and functions available through technology to calculate distances and model periodic motion; and

(C) use direct and inverse variation to describe physical laws such as Hook’s, Newton’s, and Boyle’s laws.

(M.9) The student uses algebraic and geometric models to represent patterns and structures.

The student is expected to:

(A) use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture; and

(B) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics
Subchapter D. Other High School Mathematics Courses

Statutory Authority: The provisions of this Subchapter D issued under the Texas Education Code, §28.002, unless otherwise noted.

§111.51. Implementation of Texas Essential Knowledge and Skills for Mathematics, Other High School Mathematics Courses.

The provisions of this subchapter shall be implemented by school districts beginning September 1, 1998, and at that time shall supersede §75.63(o), (q)-(u), and (cc) of this title (relating to Mathematics).

Source: The provisions of this §111.51 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.52. Independent Study in Mathematics (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of Independent Study in Mathematics. Required prerequisites: Algebra II, Geometry. Students may repeat this course with different course content for a second credit.

(b) Content requirements. Students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics, such as theory of equations, number theory, non-Euclidean geometry, advanced survey of mathematics, or history of mathematics. The requirements for each course must be approved by the local district before the course begins.

(c) If this course is being used to satisfy requirements for the Distinguished Achievement Program, student research/products must be presented before a panel of professionals or approved by the student’s mentor.

Source: The provisions of this §111.52 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.53. Advanced Placement (AP) Statistics (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Algebra II, Geometry.

(b) Content requirements. Content requirements for Advanced Placement (AP) Statistics are prescribed in the College Board Publication Advanced Placement Course Description: Statistics, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.53 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.54. Advanced Placement (AP) Calculus AB (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.

(b) Content requirements. Content requirements for Advanced Placement (AP) Calculus AB are prescribed in the College Board Publication Advanced Placement Course Description Mathematics: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.54 adopted to be effective September 1, 1998, 22 TexReg 7623.
§111.55. Advanced Placement (AP) Calculus BC (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.

(b) Content requirements. Content requirements for Advanced Placement (AP) Calculus BC are prescribed in the College Board Publication Advanced Placement Course Description: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.55 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.56. IB Mathematical Studies Subsidiary Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematical Studies Subsidiary Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: Algebra II, Geometry.

(b) Content requirements. Content requirements for IB Mathematical Studies Subsidiary Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.56 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.57. IB Mathematical Methods Subsidiary Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematical Methods Subsidiary Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisites: Algebra II, Geometry.

(b) Content requirements. Content requirements for IB Mathematical Methods Subsidiary Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.57 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.58. IB Mathematics Higher Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematics Higher Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematical Studies Subsidiary Level or IB Mathematical Methods Subsidiary Level.

(b) Content requirements. Content requirements for IB Mathematics Higher Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.58 adopted to be effective September 1, 1998, 22 TexReg 7623.
§111.59. IB Advanced Mathematics Subsidiary Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Advanced Mathematics Subsidiary Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematics Higher Level.

(b) Content requirements. Content requirements for IB Advanced Mathematics Subsidiary Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.59 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.60. Concurrent Enrollment in College Courses.

(a) General requirements. Students shall be awarded one-half credit for each semester of successful completion of a college course in which the student is concurrently enrolled while in high school.

(b) Content requirements. In order for students to receive state graduation credit for concurrent enrollment courses, content requirements must meet or exceed the essential knowledge and skills in a given course.

Source: The provisions of this §111.60 adopted to be effective September 1, 1998, 22 TexReg 7623.
About the finding aids we have added to this version of the mathematics TEKS

For the mathematics TEKS adopted in 2012, we used pdfs from the Texas Education Agency website, ritter.tea.state.tx.us/rules/tac/chapter111/index.html. The only revisions we made to these pages were to include an additional header and footer to help users distinguish between the two sets of TEKS in this book. Specifically, we added:

- Running footers on every page of the mathematics TEKS adopted in 2012 that specify “Chapter 111. Texas Essential Knowledge and Skills for Mathematics” or “2012 Adoption.”
- Running headers on alternating pages that specify “Ch. 111, TEKS for Mathematics (2012 Adoption).”

The grade grouping (elementary, middle school, high school, or other high school mathematics courses) identifiers are part of the original pdf files from the Texas Education Agency website.
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2012 Adoption

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Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter A. Elementary

Statutory Authority: The provisions of this Subchapter A issued under the Texas Education Code, §§7.102(c)(4), 28.002, and 28.0021(a)(1), unless otherwise noted.

§111.1. Implementation of Texas Essential Knowledge and Skills for Mathematics, Elementary, Adopted 2012.

(a) The provisions of §§111.2-111.7 of this subchapter shall be implemented by school districts.

(b) No later than August 31, 2013, the commissioner of education shall determine whether instructional materials funding has been made available to Texas public schools for materials that cover the essential knowledge and skills for mathematics as adopted in §§111.2-111.7 of this subchapter.

(c) If the commissioner makes the determination that instructional materials funding has been made available under subsection (b) of this section, §§111.2-111.7 of this subchapter shall be implemented beginning with the 2014-2015 school year and apply to the 2014-2015 and subsequent school years.

(d) If the commissioner does not make the determination that instructional materials funding has been made available under subsection (b) of this section, the commissioner shall determine no later than August 31 of each subsequent school year whether instructional materials funding has been made available. If the commissioner determines that instructional materials funding has been made available, the commissioner shall notify the State Board of Education and school districts that §§111.2-111.7 of this subchapter shall be implemented for the following school year.

(e) Sections 111.11-111.17 of this subchapter shall be superseded by the implementation of §§111.1-111.7 under this section.

Source: The provisions of this §111.1 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.2. Kindergarten, Adopted 2012.

(a) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Kindergarten are expected to perform their work without the use of calculators.

The primary focal areas in Kindergarten are understanding counting and cardinality, understanding addition as joining and subtraction as separating, and comparing objects by measurable attributes.

Students develop number and operations through several fundamental concepts. Students know number names and the counting sequence. Counting and cardinality lay a solid foundation for number. Students apply the principles of counting to make the connection between numbers and quantities.

Students use meanings of numbers to create strategies for solving problems and responding to practical situations involving addition and subtraction.

Students identify characteristics of objects that can be measured and directly compare objects according to these measurable attributes.

Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- apply mathematics to problems arising in everyday life, society, and the workplace;
- use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- create and use representations to organize, record, and communicate mathematical ideas;
- analyze mathematical relationships to connect and communicate mathematical ideas; and
- display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:

- count forward and backward to at least 20 with and without objects;
- read, write, and represent whole numbers from 0 to at least 20 with and without objects or pictures;
- count a set of objects up to at least 20 and demonstrate that the last number said tells the number of objects in the set regardless of their arrangement or order;
- recognize instantly the quantity of a small group of objects in organized and random arrangements;
(E) generate a set using concrete and pictorial models that represents a number that is more than, less than, and equal to a given number up to 20;
(F) generate a number that is one more than or one less than another number up to at least 20;
(G) compare sets of objects up to at least 20 in each set using comparative language;
(H) use comparative language to describe two numbers up to 20 presented as written numerals; and
(I) compose and decompose numbers up to 10 with objects and pictures.
(3) Number and operations. The student applies mathematical process standards to develop an understanding of addition and subtraction situations in order to solve problems. The student is expected to:
(A) model the action of joining to represent addition and the action of separating to represent subtraction;
(B) solve word problems using objects and drawings to find sums up to 10 and differences within 10; and
(C) explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences.
(4) Number and operations. The student applies mathematical process standards to identify coins in order to recognize the need for monetary transactions. The student is expected to identify U.S. coins by name, including pennies, nickels, dimes, and quarters.
(5) Algebraic reasoning. The student applies mathematical process standards to identify the pattern in the number word list. The student is expected to recite numbers up to at least 100 by ones and tens beginning with any given number.
(6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:
(A) identify two-dimensional shapes, including circles, triangles, rectangles, and squares as special rectangles;
(B) identify three-dimensional solids, including cylinders, cones, spheres, and cubes, in the real world;
(C) identify two-dimensional components of three-dimensional objects;
(D) identify attributes of two-dimensional shapes using informal and formal geometric language interchangeably;
(E) classify and sort a variety of regular and irregular two- and three-dimensional figures regardless of orientation or size; and
(F) create two-dimensional shapes using a variety of materials and drawings.
(7) Geometry and measurement. The student applies mathematical process standards to directly compare measurable attributes. The student is expected to:
(A) give an example of a measurable attribute of a given object, including length, capacity, and weight; and
(B) compare two objects with a common measurable attribute to see which object has more or less of the attribute and describe the difference.
(8) Data analysis. The student applies mathematical process standards to collect and organize data to make it useful for interpreting information. The student is expected to:
(A) collect, sort, and organize data into two or three categories;
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(B) use data to create real-object and picture graphs; and
(C) draw conclusions from real-object and picture graphs.

(9) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:
  (A) identify ways to earn income;
  (B) differentiate between money received as income and money received as gifts;
  (C) list simple skills required for jobs; and
  (D) distinguish between wants and needs and identify income as a source to meet one's wants and needs.

Source: The provisions of this §111.2 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.3. Grade 1, Adopted 2012.

(a) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 1 are expected to perform their work without the use of calculators.

(4) The primary focal areas in Grade 1 are understanding and applying place value, solving problems involving addition and subtraction, and composing and decomposing two-dimensional shapes and three-dimensional solids.

(A) Students use relationships within the numeration system to understand the sequential order of the counting numbers and their relative magnitude.

(B) Students extend their use of addition and subtraction beyond the actions of joining and separating to include comparing and combining. Students use properties of operations and the relationship between addition and subtraction to solve problems. By comparing a
variety of solution strategies, students use efficient, accurate, and generalizable methods to perform operations.

(C) Students use basic shapes and spatial reasoning to model objects in their environment and construct more complex shapes. Students are able to identify, name, and describe basic two-dimensional shapes and three-dimensional solids.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:

(A) recognize instantly the quantity of structured arrangements;

(B) use concrete and pictorial models to compose and decompose numbers up to 120 in more than one way as so many hundreds, so many tens, and so many ones;

(C) use objects, pictures, and expanded and standard forms to represent numbers up to 120;

(D) generate a number that is greater than or less than a given whole number up to 120;

(E) use place value to compare whole numbers up to 120 using comparative language;

(F) order whole numbers up to 120 using place value and open number lines; and

(G) represent the comparison of two numbers to 100 using the symbols >, <, or =.

(3) Number and operations. The student applies mathematical process standards to develop and use strategies for whole number addition and subtraction computations in order to solve problems. The student is expected to:

(A) use concrete and pictorial models to determine the sum of a multiple of 10 and a one-digit number in problems up to 99;

(B) use objects and pictorial models to solve word problems involving joining, separating, and comparing sets within 20 and unknowns as any one of the terms in the problem such as 2 + 4 = [ ]; 3 + [ ] = 7; and 5 = [ ] - 3;

(C) compose 10 with two or more addends with and without concrete objects;
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(D) apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a 10;

(E) explain strategies used to solve addition and subtraction problems up to 20 using spoken words, objects, pictorial models, and number sentences; and

(F) generate and solve problem situations when given a number sentence involving addition or subtraction of numbers within 20.

(4) Number and operations. The student applies mathematical process standards to identify coins, their values, and the relationships among them in order to recognize the need for monetary transactions. The student is expected to:

(A) identify U.S. coins, including pennies, nickels, dimes, and quarters, by value and describe the relationships among them;

(B) write a number with the cent symbol to describe the value of a coin; and

(C) use relationships to count by twos, fives, and tens to determine the value of a collection of pennies, nickels, and/or dimes.

(5) Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:

(A) recite numbers forward and backward from any given number between 1 and 120;

(B) skip count by twos, fives, and tens to determine the total number of objects up to 120 in a set;

(C) use relationships to determine the number that is 10 more and 10 less than a given number up to 120;

(D) represent word problems involving addition and subtraction of whole numbers up to 20 using concrete and pictorial models and number sentences;

(E) understand that the equal sign represents a relationship where expressions on each side of the equal sign represent the same value(s);

(F) determine the unknown whole number in an addition or subtraction equation when the unknown may be any one of the three or four terms in the equation; and

(G) apply properties of operations to add and subtract two or three numbers.

(6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:

(A) classify and sort regular and irregular two-dimensional shapes based on attributes using informal geometric language;

(B) distinguish between attributes that define a two-dimensional or three-dimensional figure and attributes that do not define the shape;

(C) create two-dimensional figures, including circles, triangles, rectangles, and squares, as special rectangles, rhombuses, and hexagons;

(D) identify two-dimensional shapes, including circles, triangles, rectangles, and squares, as special rectangles, rhombuses, and hexagons and describe their attributes using formal geometric language;

(E) identify three-dimensional solids, including spheres, cones, cylinders, rectangular prisms (including cubes), and triangular prisms, and describe their attributes using formal geometric language;
(F) compose two-dimensional shapes by joining two, three, or four figures to produce a
    target shape in more than one way if possible;

(G) partition two-dimensional figures into two and four fair shares or equal parts and describe
    the parts using words; and

(H) identify examples and non-examples of halves and fourths.

(7) Geometry and measurement. The student applies mathematical process standards to select and use
    units to describe length and time. The student is expected to:

(A) use measuring tools to measure the length of objects to reinforce the continuous nature of
    linear measurement;

(B) illustrate that the length of an object is the number of same-size units of length that, when
    laid end-to-end with no gaps or overlaps, reach from one end of the object to the other;

(C) measure the same object/distance with units of two different lengths and describe how
    and why the measurements differ;

(D) describe a length to the nearest whole unit using a number and a unit; and

(E) tell time to the hour and half hour using analog and digital clocks.

(8) Data analysis. The student applies mathematical process standards to organize data to make it
    useful for interpreting information and solving problems. The student is expected to:

(A) collect, sort, and organize data in up to three categories using models/representations
    such as tally marks or T-charts;

(B) use data to create picture and bar-type graphs; and

(C) draw conclusions and generate and answer questions using information from picture and
    bar-type graphs.

(9) Personal financial literacy. The student applies mathematical process standards to manage one's
    financial resources effectively for lifetime financial security. The student is expected to:

(A) define money earned as income;

(B) identify income as a means of obtaining goods and services, oftentimes making choices
    between wants and needs;

(C) distinguish between spending and saving; and

(D) consider charitable giving.

Source: The provisions of this §111.3 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.4. Grade 2, Adopted 2012.

(a) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential
    knowledge and skills for mathematics, guided by the college and career readiness standards. By
    embedding statistics, probability, and finance, while focusing on computational thinking,
    mathematical fluency, and solid understanding, Texas will lead the way in mathematics education
    and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The
    placement of the process standards at the beginning of the knowledge and skills listed for each
    grade and course is intentional. The process standards weave the other knowledge and skills
    together so that students may be successful problem solvers and use mathematics efficiently and
    effectively in daily life. The process standards are integrated at every grade level and course.
    When possible, students will apply mathematics to problems arising in everyday life, society, and
    the workplace. Students will use a problem-solving model that incorporates analyzing given
information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 2 are expected to perform their work without the use of calculators.

(4) The primary focal areas in Grade 2 are making comparisons within the base-10 place value system, solving problems with addition and subtraction within 1,000, and building foundations for multiplication.

(A) Students develop an understanding of the base-10 place value system and place value concepts. The students' understanding of base-10 place value includes ideas of counting in units and multiples of thousands, hundreds, tens, and ones and a grasp of number relationships, which students demonstrate in a variety of ways.

(B) Students identify situations in which addition and subtraction are useful to solve problems. Students develop a variety of strategies to use efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers.

(C) Students use the relationship between skip counting and equal groups of objects to represent the addition or subtraction of equivalent sets, which builds a strong foundation for multiplication and division.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:

(A) use concrete and pictorial models to compose and decompose numbers up to 1,200 in more than one way as a sum of so many thousands, hundreds, tens, and ones;

(B) use standard, word, and expanded forms to represent numbers up to 1,200;

(C) generate a number that is greater than or less than a given whole number up to 1,200;

(D) use place value to compare and order whole numbers up to 1,200 using comparative language, numbers, and symbols (> , <, or =);

(E) locate the position of a given whole number on an open number line; and

(F) name the whole number that corresponds to a specific point on a number line.

(3) Number and operations. The student applies mathematical process standards to recognize and represent fractional units and communicates how they are used to name parts of a whole. The student is expected to:

(A) partition objects into equal parts and name the parts, including halves, fourths, and eighths, using words;

(B) explain that the more fractional parts used to make a whole, the smaller the part; and the fewer the fractional parts, the larger the part;

(C) use concrete models to count fractional parts beyond one whole using words and recognize how many parts it takes to equal one whole; and

(D) identify examples and non-examples of halves, fourths, and eighths.

(4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve addition and subtraction problems with efficiency and accuracy. The student is expected to:

(A) recall basic facts to add and subtract within 20 with automaticity;

(B) add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations;

(C) solve one-step and multi-step word problems involving addition and subtraction within 1,000 using a variety of strategies based on place value, including algorithms; and

(D) generate and solve problem situations for a given mathematical number sentence involving addition and subtraction of whole numbers within 1,000.

(5) Number and operations. The student applies mathematical process standards to determine the value of coins in order to solve monetary transactions. The student is expected to:

(A) determine the value of a collection of coins up to one dollar; and

(B) use the cent symbol, dollar sign, and the decimal point to name the value of a collection of coins.

(6) Number and operations. The student applies mathematical process standards to connect repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. The student is expected to:

(A) model, create, and describe contextual multiplication situations in which equivalent sets of concrete objects are joined; and

(B) model, create, and describe contextual division situations in which a set of concrete objects is separated into equivalent sets.
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(7) Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:

(A) determine whether a number up to 40 is even or odd using pairings of objects to represent the number;

(B) use an understanding of place value to determine the number that is 10 or 100 more or less than a given number up to 1,200; and

(C) represent and solve addition and subtraction word problems where unknowns may be any one of the terms in the problem.

(8) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:

(A) create two-dimensional shapes based on given attributes, including number of sides and vertices;

(B) classify and sort three-dimensional solids, including spheres, cones, cylinders, rectangular prisms (including cubes as special rectangular prisms), and triangular prisms, based on attributes using formal geometric language;

(C) classify and sort polygons with 12 or fewer sides according to attributes, including identifying the number of sides and number of vertices;

(D) compose two-dimensional shapes and three-dimensional solids with given properties or attributes; and

(E) decompose two-dimensional shapes such as cutting out a square from a rectangle, dividing a shape in half, or partitioning a rectangle into identical triangles and identify the resulting geometric parts.

(9) Geometry and measurement. The student applies mathematical process standards to select and use units to describe length, area, and time. The student is expected to:

(A) find the length of objects using concrete models for standard units of length;

(B) describe the inverse relationship between the size of the unit and the number of units needed to equal the length of an object;

(C) represent whole numbers as distances from any given location on a number line;

(D) determine the length of an object to the nearest marked unit using rulers, yardsticks, meter sticks, or measuring tapes;

(E) determine a solution to a problem involving length, including estimating lengths;

(F) use concrete models of square units to find the area of a rectangle by covering it with no gaps or overlaps, counting to find the total number of square units, and describing the measurement using a number and the unit; and

(G) read and write time to the nearest one-minute increment using analog and digital clocks and distinguish between a.m. and p.m.

(10) Data analysis. The student applies mathematical process standards to organize data to make it useful for interpreting information and solving problems. The student is expected to:

(A) explain that the length of a bar in a bar graph or the number of pictures in a pictograph represents the number of data points for a given category;

(B) organize a collection of data with up to four categories using pictographs and bar graphs with intervals of one or more;
(C) write and solve one-step word problems involving addition or subtraction using data represented within pictographs and bar graphs with intervals of one; and

(D) draw conclusions and make predictions from information in a graph.

(11) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

(A) calculate how money saved can accumulate into a larger amount over time;

(B) explain that saving is an alternative to spending;

(C) distinguish between a deposit and a withdrawal;

(D) identify examples of borrowing and distinguish between responsible and irresponsible borrowing;

(E) identify examples of lending and use concepts of benefits and costs to evaluate lending decisions; and

(F) differentiate between producers and consumers and calculate the cost to produce a simple item.

Source: The provisions of this §111.4 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.5. Grade 3, Adopted 2012.

(a) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 3 are expected to perform their work without the use of calculators.

(4) The primary focal areas in Grade 3 are place value, operations of whole numbers, and understanding fractional units. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data...
§111.A.

analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will focus on applying place value, comparing and ordering whole numbers, connecting multiplication and division, and understanding and representing fractions as numbers and equivalent fractions. In algebraic reasoning, students will use multiple representations of problem situations, determine missing values in number sentences, and represent real-world relationships using number pairs in a table and verbal descriptions. In geometry and measurement, students will identify and classify two-dimensional figures according to common attributes, decompose composite figures formed by rectangles to determine area, determine the perimeter of polygons, solve problems involving time, and measure liquid volume (capacity) or weight. In data analysis, students will represent and interpret data.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;
(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
(E) create and use representations to organize, record, and communicate mathematical ideas;
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:

(A) compose and decompose numbers up to 100,000 as a sum of so many ten thousands, so many thousands, so many hundreds, so many tens, and so many ones using objects, pictorial models, and numbers, including expanded notation as appropriate;
(B) describe the mathematical relationships found in the base-10 place value system through the hundred thousands place;
(C) represent a number on a number line as being between two consecutive multiples of 10; 100; 1,000; or 10,000 and use words to describe relative size of numbers in order to round whole numbers; and
(D) compare and order whole numbers up to 100,000 and represent comparisons using the symbols >, <, or =.

(3) Number and operations. The student applies mathematical process standards to represent and explain fractional units. The student is expected to:

(A) represent fractions greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 using concrete objects and pictorial models, including strip diagrams and number lines;
(B) determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 given a specified point on a number line;

(C) explain that the unit fraction $\frac{1}{b}$ represents the quantity formed by one part of a whole that has been partitioned into $b$ equal parts where $b$ is a non-zero whole number;

(D) compose and decompose a fraction $\frac{a}{b}$ with a numerator greater than zero and less than or equal to $b$ as a sum of parts $\frac{1}{b}$;

(E) solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8;

(F) represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines;

(G) explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model; and

(H) compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models.

(4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:

(A) solve with fluency one-step and two-step problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations, and the relationship between addition and subtraction;

(B) round to the nearest 10 or 100 or use compatible numbers to estimate solutions to addition and subtraction problems;

(C) determine the value of a collection of coins and bills;

(D) determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays up to 10 by 10;

(E) represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting;

(F) recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division facts;

(G) use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;

(H) determine the number of objects in each group when a set of objects is partitioned into equal shares or a set of objects is shared equally;

(I) determine if a number is even or odd using divisibility rules;

(J) determine a quotient using the relationship between multiplication and division; and

(K) solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts.
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(A) represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations;

(B) represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations;

(C) describe a multiplication expression as a comparison such as 3 x 24 represents 3 times as much as 24;

(D) determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product; and

(E) represent real-world relationships using number pairs in a table and verbal descriptions.

(6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:

(A) classify and sort two- and three-dimensional solids, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes, based on attributes using formal geometric language;

(B) use attributes to recognize rhombuses, parallelograms, trapezoids, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories;

(C) determine the area of rectangles with whole number side lengths in problems using multiplication related to the number of rows times the number of unit squares in each row;

(D) decompose composite figures formed by rectangles into non-overlapping rectangles to determine the area of the original figure using the additive property of area; and

(E) decompose two congruent two-dimensional figures into parts with equal areas and express the area of each part as a unit fraction of the whole and recognize that equal shares of identical wholes need not have the same shape.

(7) Geometry and measurement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary and metric measurement. The student is expected to:

(A) represent fractions of halves, fourths, and eighths as distances from zero on a number line;

(B) determine the perimeter of a polygon or a missing length when given perimeter and remaining side lengths in problems;

(C) determine the solutions to problems involving addition and subtraction of time intervals in minutes using pictorial models or tools such as a 15-minute event plus a 30-minute event equals 45 minutes;

(D) determine when it is appropriate to use measurements of liquid volume (capacity) or weight; and

(E) determine liquid volume (capacity) or weight using appropriate units and tools.

(8) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:

(A) summarize a data set with multiple categories using a frequency table, dot plot, pictograph, or bar graph with scaled intervals; and

(B) solve one- and two-step problems using categorical data represented with a frequency table, dot plot, pictograph, or bar graph with scaled intervals.
(9) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

(A) explain the connection between human capital/labor and income;

(B) describe the relationship between the availability or scarcity of resources and how that impacts cost;

(C) identify the costs and benefits of planned and unplanned spending decisions;

(D) explain that credit is used when wants or needs exceed the ability to pay and that it is the borrower's responsibility to pay it back to the lender, usually with interest;

(E) list reasons to save and explain the benefit of a savings plan, including for college; and

(F) identify decisions involving income, spending, saving, credit, and charitable giving.

Source: The provisions of this §111.5 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.6. Grade 4, Adopted 2012.

(a) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 4 are expected to perform their work without the use of calculators.

(4) The primary focal areas in Grade 4 are use of operations, fractions, and decimals and describing and analyzing geometry and measurement. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will apply place value and represent points on a number line that correspond to a given fraction or terminating decimal. In algebraic reasoning, students will represent and solve multi-step problems involving the four operations with whole numbers with expressions and equations and generate and analyze patterns. In geometry and measurement,
students will classify two-dimensional figures, measure angles, and convert units of measure. In data analysis, students will represent and interpret data.

(5) Statements that contain the word “including” reference content that must be mastered, while those containing the phrase “such as” are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;
(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
(E) create and use representations to organize, record, and communicate mathematical ideas;
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent, compare, and order whole numbers and decimals and understand relationships related to place value. The student is expected to:

(A) interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left;
(B) represent the value of the digit in whole numbers through 1,000,000,000 and decimals to the hundredths using expanded notation and numerals;
(C) compare and order whole numbers to 1,000,000,000 and represent comparisons using the symbols >, <, or =;
(D) round whole numbers to a given place value through the hundred thousands place;
(E) represent decimals, including tenths and hundredths, using concrete and visual models and money;
(F) compare and order decimals using concrete and visual models to the hundredths;
(G) relate decimals to fractions that name tenths and hundredths; and
(H) determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line.

(3) Number and operations. The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to:

(A) represent a fraction \( \frac{a}{b} \) as a sum of fractions \( \frac{1}{b} \), where \( a \) and \( b \) are whole numbers and \( b > 0 \), including when \( a > b \);
(B) decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations;
(C) determine if two given fractions are equivalent using a variety of methods;
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(D) compare two fractions with different numerators and different denominators and represent the comparison using the symbols >, =, or <;

(E) represent and solve addition and subtraction of fractions with equal denominators using objects and pictorial models that build to the number line and properties of operations;

(F) evaluate the reasonableness of sums and differences of fractions using benchmark fractions 0, 1/4, 1/2, 3/4, and 1, referring to the same whole; and

(G) represent fractions and decimals to the tenths or hundredths as distances from zero on a number line.

(4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations and decimal sums and differences in order to solve problems with efficiency and accuracy. The student is expected to:

(A) add and subtract whole numbers and decimals to the hundredths place using the standard algorithm;

(B) determine products of a number and 10 or 100 using properties of operations and place value understandings;

(C) represent the product of 2 two-digit numbers using arrays, area models, or equations, including perfect squares through 15 by 15;

(D) use strategies and algorithms, including the standard algorithm, to multiply up to a four-digit number by a one-digit number and to multiply a two-digit number by a two-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;

(E) represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations;

(F) use strategies and algorithms, including the standard algorithm, to divide up to a four-digit dividend by a one-digit divisor;

(G) round to the nearest 10, 100, or 1,000 or use compatible numbers to estimate solutions involving whole numbers; and

(H) solve with fluency one- and two-step problems involving multiplication and division, including interpreting remainders.

(5) Algebraic reasoning. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(A) represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity;

(B) represent problems using an input-output table and numerical expressions to generate a number pattern that follows a given rule representing the relationship of the values in the resulting sequence and their position in the sequence;

(C) use models to determine the formulas for the perimeter of a rectangle \((l + w + l + w \text{ or } 2l + 2w)\), including the special form for perimeter of a square \((4s)\) and the area of a rectangle \((l \times w)\); and

(D) solve problems related to perimeter and area of rectangles where dimensions are whole numbers.

(6) Geometry and measurement. The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

(A) identify points, lines, line segments, rays, angles, and perpendicular and parallel lines;
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(B) identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure;
(C) apply knowledge of right angles to identify acute, right, and obtuse triangles; and
(D) classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

(7) Geometry and measurement. The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:
   (A) illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle. Angle measures are limited to whole numbers;
   (B) illustrate degrees as the units used to measure an angle, where 1/360 of any circle is one degree and an angle that "cuts" n/360 out of any circle whose center is at the angle's vertex has a measure of n degrees. Angle measures are limited to whole numbers;
   (C) determine the approximate measures of angles in degrees to the nearest whole number using a protractor;
   (D) draw an angle with a given measure; and
   (E) determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures.

(8) Geometry and measurement. The student applies mathematical process standards to select appropriate customary and metric units, strategies, and tools to solve problems involving measurement. The student is expected to:
   (A) identify relative sizes of measurement units within the customary and metric systems;
   (B) convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table; and
   (C) solve problems that deal with measurements of length, intervals of time, liquid volumes, mass, and money using addition, subtraction, multiplication, or division as appropriate.

(9) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:
   (A) represent data on a frequency table, dot plot, or stem-and-leaf plot marked with whole numbers and fractions; and
   (B) solve one- and two-step problems using data in whole number, decimal, and fraction form in a frequency table, dot plot, or stem-and-leaf plot.

(10) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:
   (A) distinguish between fixed and variable expenses;
   (B) calculate profit in a given situation;
   (C) compare the advantages and disadvantages of various savings options;
   (D) describe how to allocate a weekly allowance among spending, saving, including for college; and sharing; and
   (E) describe the basic purpose of financial institutions, including keeping money safe, borrowing money, and lending.

Source: The provisions of this §111.6 adopted to be effective September 10, 2012, 37 TexReg 7109.
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§111.7. Grade 5, Adopted 2012.

(a)  Introduction.

   (1)  The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

   (2)  The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

   (3)  For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 5 are expected to perform their work without the use of calculators.

   (4)  The primary focal areas in Grade 5 are solving problems involving all four operations with positive rational numbers, determining and generating formulas and solutions to expressions, and extending measurement to area and volume. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will apply place value and identify part-to-whole relationships and equivalence. In algebraic reasoning, students will represent and solve problems with expressions and equations, build foundations of functions through patterning, identify prime and composite numbers, and use the order of operations. In geometry and measurement, students will classify two-dimensional figures, connect geometric attributes to the measures of three-dimensional figures, use units of measure, and represent location using a coordinate plane. In data analysis, students will represent and interpret data.

   (5)  Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b)  Knowledge and skills.

   (1)  Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

      (A)  apply mathematics to problems arising in everyday life, society, and the workplace;
      (B)  use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
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(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent, compare, and order positive rational numbers and understand relationships as related to place value. The student is expected to:

(A) represent the value of the digit in decimals through the thousandths using expanded notation and numerals;

(B) compare and order two decimals to thousandths and represent comparisons using the symbols >, <, or =; and

(C) round decimals to tenths or hundredths.

(3) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:

(A) estimate to determine solutions to mathematical and real-world problems involving addition, subtraction, multiplication, or division;

(B) multiply with fluency a three-digit number by a two-digit number using the standard algorithm;

(C) solve with proficiency for quotients of up to a four-digit dividend by a two-digit divisor using strategies and the standard algorithm;

(D) represent multiplication of decimals with products to the hundredths using objects and pictorial models, including area models;

(E) solve for products of decimals to the hundredths, including situations involving money, using strategies based on place-value understandings, properties of operations, and the relationship to the multiplication of whole numbers;

(F) represent quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using objects and pictorial models, including area models;

(G) solve for quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using strategies and algorithms, including the standard algorithm;

(H) represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models and properties of operations;

(I) represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models;

(J) represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as $1/3 ÷ 7$ and $7 ÷ 1/3$ using objects and pictorial models, including area models;

(K) add and subtract positive rational numbers fluently; and

(L) divide whole numbers by unit fractions and unit fractions by whole numbers.
(4) Algebraic reasoning. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(A) identify prime and composite numbers;
(B) represent and solve multi-step problems involving the four operations with whole numbers using equations with a letter standing for the unknown quantity;
(C) generate a numerical pattern when given a rule in the form $y = ax$ or $y = x + a$ and graph;
(D) recognize the difference between additive and multiplicative numerical patterns given in a table or graph;
(E) describe the meaning of parentheses and brackets in a numeric expression;
(F) simplify numerical expressions that do not involve exponents, including up to two levels of grouping;
(G) use concrete objects and pictorial models to develop the formulas for the volume of a rectangular prism, including the special form for a cube ($V = l \times w \times h$, $V = s \times s \times s$, and $V = Bh$); and
(H) represent and solve problems related to perimeter and/or area and related to volume.

(5) Geometry and measurement. The student applies mathematical process standards to classify two-dimensional figures by attributes and properties. The student is expected to classify two-dimensional figures in a hierarchy of sets and subsets using graphic organizers based on their attributes and properties.

(6) Geometry and measurement. The student applies mathematical process standards to understand, recognize, and quantify volume. The student is expected to:

(A) recognize a cube with side length of one unit as a unit cube having one cubic unit of volume and the volume of a three-dimensional figure as the number of unit cubes ($n$ cubic units) needed to fill it with no gaps or overlaps if possible; and
(B) determine the volume of a rectangular prism with whole number side lengths in problems related to the number of layers times the number of unit cubes in the area of the base.

(7) Geometry and measurement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving measurement. The student is expected to solve problems by calculating conversions within a measurement system, customary or metric.

(8) Geometry and measurement. The student applies mathematical process standards to identify locations on a coordinate plane. The student is expected to:

(A) describe the key attributes of the coordinate plane, including perpendicular number lines (axes) where the intersection (origin) of the two lines coincides with zero on each number line and the given point (0, 0); the $x$-coordinate, the first number in an ordered pair, indicates movement parallel to the $x$-axis starting at the origin; and the $y$-coordinate, the second number, indicates movement parallel to the $y$-axis starting at the origin;
(B) describe the process for graphing ordered pairs of numbers in the first quadrant of the coordinate plane; and
(C) graph in the first quadrant of the coordinate plane ordered pairs of numbers arising from mathematical and real-world problems, including those generated by number patterns or found in an input-output table.

(9) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:
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(A) represent categorical data with bar graphs or frequency tables and numerical data, including data sets of measurements in fractions or decimals, with dot plots or stem-and-leaf plots;
(B) represent discrete paired data on a scatterplot; and
(C) solve one- and two-step problems using data from a frequency table, dot plot, bar graph, stem-and-leaf plot, or scatterplot.

(10) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:
(A) define income tax, payroll tax, sales tax, and property tax;
(B) explain the difference between gross income and net income;
(C) identify the advantages and disadvantages of different methods of payment, including check, credit card, debit card, and electronic payments;
(D) develop a system for keeping and using financial records;
(E) describe actions that might be taken to balance a budget when expenses exceed income; and
(F) balance a simple budget.

Source: The provisions of this §111.7 adopted to be effective September 10, 2012, 37 TexReg 7109.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter B. Middle School

Statutory Authority: The provisions of this Subchapter B issued under the Texas Education Code, §§7.102(c)(4), 28.002, 28.0021(a)(1), and 28.008, unless otherwise noted.
§111.25. Implementation of Texas Essential Knowledge and Skills for Mathematics, Middle School, Adopted 2012.

(a) The provisions of §§111.26-111.28 of this subchapter shall be implemented by school districts.

(b) No later than August 31, 2013, the commissioner of education shall determine whether instructional materials funding has been made available to Texas public schools for materials that cover the essential knowledge and skills for mathematics as adopted in §§111.26-111.28 of this subchapter.

(c) If the commissioner makes the determination that instructional materials funding has been made available under subsection (b) of this section, §§111.26-111.28 of this subchapter shall be implemented beginning with the 2014-2015 school year and apply to the 2014-2015 and subsequent school years.

(d) If the commissioner does not make the determination that instructional materials funding has been made available under subsection (b) of this section, the commissioner shall determine no later than August 31 of each subsequent school year whether instructional materials funding has been made available. If the commissioner determines that instructional materials funding has been made available, the commissioner shall notify the State Board of Education and school districts that §§111.26-111.28 of this subchapter shall be implemented for the following school year.

(e) Sections 111.21-111.24 of this subchapter shall be superseded by the implementation of §§111.25-111.28 under this section.

Source: The provisions of this §111.25 adopted to be effective September 10, 2012, 37 TexReg 7109.


(a) Introduction.

   (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

   (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

   (3) The primary focal areas in Grade 6 are number and operations; proportionality; expressions, equations, and relationships; and measurement and data. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students use geometric properties and relationships, as well as spatial reasoning, to
model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.

(4) Statements that contain the word “including” reference content that must be mastered, while those containing the phrase “such as” are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;
(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
(E) create and use representations to organize, record, and communicate mathematical ideas;
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to:

(A) classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers;
(B) identify a number, its opposite, and its absolute value;
(C) locate, compare, and order integers and rational numbers using a number line;
(D) order a set of rational numbers arising from mathematical and real-world contexts; and
(E) extend representations for division to include fraction notation such as $a/b$ represents the same number as $a \div b$ where $b \neq 0$.

(3) Number and operations. The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:

(A) recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values;
(B) determine, with and without computation, whether a quantity is increased or decreased when multiplied by a fraction, including values greater than or less than one;
(C) represent integer operations with concrete models and connect the actions with the models to standardized algorithms;
(D) add, subtract, multiply, and divide integers fluently; and
(E) multiply and divide positive rational numbers fluently.
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(4) Proportionality. The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to:

(A) compare two rules verbally, numerically, graphically, and symbolically in the form of \( y = ax \) or \( y = x + a \) in order to differentiate between additive and multiplicative relationships;

(B) apply qualitative and quantitative reasoning to solve prediction and comparison of real-world problems involving ratios and rates;

(C) give examples of ratios as multiplicative comparisons of two quantities describing the same attribute;

(D) give examples of rates as the comparison by division of two quantities having different attributes, including rates as quotients;

(E) represent ratios and percents with concrete models, fractions, and decimals;

(F) represent benchmark fractions and percents such as 1%, 10%, 25%, 33 1/3%, and multiples of these values using 10 by 10 grids, strip diagrams, number lines, and numbers;

(G) generate equivalent forms of fractions, decimals, and percents using real-world problems, including problems that involve money; and

(H) convert units within a measurement system, including the use of proportions and unit rates.

(5) Proportionality. The student applies mathematical process standards to solve problems involving proportional relationships. The student is expected to:

(A) represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions;

(B) solve real-world problems to find the whole given a part and the percent, to find the part given the whole and the percent, and to find the percent given the part and the whole, including the use of concrete and pictorial models; and

(C) use equivalent fractions, decimals, and percents to show equal parts of the same whole.

(6) Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to describe algebraic relationships. The student is expected to:

(A) identify independent and dependent quantities from tables and graphs;

(B) write an equation that represents the relationship between independent and dependent quantities from a table; and

(C) represent a given situation using verbal descriptions, tables, graphs, and equations in the form \( y = kx \) or \( y = x + b \).

(7) Expressions, equations, and relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(A) generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization;

(B) distinguish between expressions and equations verbally, numerically, and algebraically;

(C) determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations; and

(D) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties.

(8) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:
(A) extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle;

(B) model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes;

(C) write equations that represent problems related to the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers; and

(D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

(9) Expressions, equations, and relationships. The student applies mathematical process standards to use equations and inequalities to represent situations. The student is expected to:

(A) write one-variable, one-step equations and inequalities to represent constraints or conditions within problems;

(B) represent solutions for one-variable, one-step equations and inequalities on number lines; and

(C) write corresponding real-world problems given one-variable, one-step equations or inequalities.

(10) Expressions, equations, and relationships. The student applies mathematical process standards to use equations and inequalities to solve problems. The student is expected to:

(A) model and solve one-variable, one-step equations and inequalities that represent problems, including geometric concepts; and

(B) determine if the given value(s) make(s) one-variable, one-step equations or inequalities true.

(11) Measurement and data. The student applies mathematical process standards to use coordinate geometry to identify locations on a plane. The student is expected to graph points in all four quadrants using ordered pairs of rational numbers.

(12) Measurement and data. The student applies mathematical process standards to use numerical or graphical representations to analyze problems. The student is expected to:

(A) represent numeric data graphically, including dot plots, stem-and-leaf plots, histograms, and box plots;

(B) use the graphical representation of numeric data to describe the center, spread, and shape of the data distribution;

(C) summarize numeric data with numerical summaries, including the mean and median (measures of center) and the range and interquartile range (IQR) (measures of spread), and use these summaries to describe the center, spread, and shape of the data distribution; and

(D) summarize categorical data with numerical and graphical summaries, including the mode, the percent of values in each category (relative frequency table), and the percent bar graph, and use these summaries to describe the data distribution.

(13) Measurement and data. The student applies mathematical process standards to use numerical or graphical representations to solve problems. The student is expected to:

(A) interpret numeric data summarized in dot plots, stem-and-leaf plots, histograms, and box plots; and

(B) distinguish between situations that yield data with and without variability.
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(14) Personal financial literacy. The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor. The student is expected to:

(A) compare the features and costs of a checking account and a debit card offered by different local financial institutions;
(B) distinguish between debit cards and credit cards;
(C) balance a check register that includes deposits, withdrawals, and transfers;
(D) explain why it is important to establish a positive credit history;
(E) describe the information in a credit report and how long it is retained;
(F) describe the value of credit reports to borrowers and to lenders;
(G) explain various methods to pay for college, including through savings, grants, scholarships, student loans, and work-study; and
(H) compare the annual salary of several occupations requiring various levels of post-secondary education or vocational training and calculate the effects of the different annual salaries on lifetime income.

Source: The provisions of this §111.26 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.27. Grade 7, Adopted 2012.

(a) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) The primary focal areas in Grade 7 are number and operations; proportionality; expressions, equations, and relationships; and measurement and data. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships, including number, geometry and measurement, and statistics and probability. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students use
geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;
(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
(E) create and use representations to organize, record, and communicate mathematical ideas;
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of rational numbers.

(3) Number and operations. The student applies mathematical process standards to add, subtract, multiply, and divide while solving problems and justifying solutions. The student is expected to:

(A) add, subtract, multiply, and divide rational numbers fluently; and
(B) apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers.

(4) Proportionality. The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:

(A) represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including \( d = rt \);
(B) calculate unit rates from rates in mathematical and real-world problems;
(C) determine the constant of proportionality \( (k = y/x) \) within mathematical and real-world problems;
(D) solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems; and
(E) convert between measurement systems, including the use of proportions and the use of unit rates.

(5) Proportionality. The student applies mathematical process standards to use geometry to describe or solve problems involving proportional relationships. The student is expected to:

(A) generalize the critical attributes of similarity, including ratios within and between similar shapes;

(B) describe π as the ratio of the circumference of a circle to its diameter; and

(C) solve mathematical and real-world problems involving similar shape and scale drawings.

(6) Proportionality. The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships. The student is expected to:

(A) represent sample spaces for simple and compound events using lists and tree diagrams;

(B) select and use different simulations to represent simple and compound events with and without technology;

(C) make predictions and determine solutions using experimental data for simple and compound events;

(D) make predictions and determine solutions using theoretical probability for simple and compound events;

(E) find the probabilities of a simple event and its complement and describe the relationship between the two;

(F) use data from a random sample to make inferences about a population;

(G) solve problems using data represented in bar graphs, dot plots, and circle graphs, including part-to-whole and part-to-part comparisons and equivalents;

(H) solve problems using qualitative and quantitative predictions and comparisons from simple experiments; and

(I) determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.

(7) Expressions, equations, and relationships. The student applies mathematical process standards to represent linear relationships using multiple representations. The student is expected to represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form \( y = mx + b \).

(8) Expressions, equations, and relationships. The student applies mathematical process standards to develop geometric relationships with volume. The student is expected to:

(A) model the relationship between the volume of a rectangular prism and a rectangular pyramid having both congruent bases and heights and connect that relationship to the formulas;

(B) explain verbally and symbolically the relationship between the volume of a triangular prism and a triangular pyramid having both congruent bases and heights and connect that relationship to the formulas; and

(C) use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas.

(9) Expressions, equations, and relationships. The student applies mathematical process standards to solve geometric problems. The student is expected to:

(A) solve problems involving the volume of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids;
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(B) determine the circumference and area of circles; 
(C) determine the area of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles; and 
(D) solve problems involving the lateral and total surface area of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape's net. 

(10) Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations and inequalities to represent situations. The student is expected to: 
   (A) write one-variable, two-step equations and inequalities to represent constraints or conditions within problems; 
   (B) represent solutions for one-variable, two-step equations and inequalities on number lines; and 
   (C) write a corresponding real-world problem given a one-variable, two-step equation or inequality. 

(11) Expressions, equations, and relationships. The student applies mathematical process standards to solve one-variable equations and inequalities. The student is expected to: 
   (A) model and solve one-variable, two-step equations and inequalities; 
   (B) determine if the given value(s) make(s) one-variable, two-step equations and inequalities true; and 
   (C) write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships. 

(12) Measurement and data. The student applies mathematical process standards to use statistical representations to analyze data. The student is expected to: 
   (A) compare two groups of numeric data using comparative dot plots or box plots by comparing their shapes, centers, and spreads; 
   (B) use data from a random sample to make inferences about a population; and 
   (C) compare two populations based on data in random samples from these populations, including informal comparative inferences about differences between the two populations. 

(13) Personal financial literacy. The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor. The student is expected to: 
   (A) calculate the sales tax for a given purchase and calculate income tax for earned wages; 
   (B) identify the components of a personal budget, including income; planned savings for college, retirement, and emergencies; taxes; and fixed and variable expenses, and calculate what percentage each category comprises of the total budget; 
   (C) create and organize a financial assets and liabilities record and construct a net worth statement; 
   (D) use a family budget estimator to determine the minimum household budget and average hourly wage needed for a family to meet its basic needs in the student's city or another large city nearby; 
   (E) calculate and compare simple interest and compound interest earnings; and 
   (F) analyze and compare monetary incentives, including sales, rebates, and coupons. 

Source: The provisions of this §111.27 adopted to be effective September 10, 2012, 37 TexReg 7109.
§111.28. Grade 8, Adopted 2012.

(a) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) The primary focal areas in Grade 8 are proportionality; expressions, equations, relationships, and foundations of functions; and measurement and data. Students use concepts, algorithms, and properties of real numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students begin to develop an understanding of functional relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
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(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:

(A) extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers;

(B) approximate the value of an irrational number, including $\pi$ and square roots of numbers less than 225, and locate that rational number approximation on a number line;

(C) convert between standard decimal notation and scientific notation; and

(D) order a set of real numbers arising from mathematical and real-world contexts.

(3) Proportionality. The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to:

(A) generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation;

(B) compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane; and

(C) use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation.

(4) Proportionality. The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:

(A) use similar right triangles to develop an understanding that slope, $m$, given as the rate comparing the change in $y$-values to the change in $x$-values, $(y_2 - y_1)/(x_2 - x_1)$, is the same for any two points $(x_1, y_1)$ and $(x_2, y_2)$ on the same line;

(B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship; and

(C) use data from a table or graph to determine the rate of change or slope and $y$-intercept in mathematical and real-world problems.

(5) Proportionality. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

(A) represent linear proportional situations with tables, graphs, and equations in the form of $y = kx$;

(B) represent linear non-proportional situations with tables, graphs, and equations in the form of $y = mx + b$, where $b \neq 0$;

(C) contrast bivariate sets of data that suggest a linear relationship with bivariate sets of data that do not suggest a linear relationship from a graphical representation;

(D) use a trend line that approximates the linear relationship between bivariate sets of data to make predictions;

(E) solve problems involving direct variation;
§111.B. Middle School

(F) distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form \( y = kx \) or \( y = mx + b \), where \( b \neq 0 \);

(G) identify functions using sets of ordered pairs, tables, mappings, and graphs;

(H) identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems; and

(I) write an equation in the form \( y = mx + b \) to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations.

(6) Expressions, equations, and relationships. The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas. The student is expected to:

(A) describe the volume formula \( V = Bh \) of a cylinder in terms of its base area and its height;

(B) model the relationship between the volume of a cylinder and a cone having both congruent bases and heights and connect that relationship to the formulas; and

(C) use models and diagrams to explain the Pythagorean theorem.

(7) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:

(A) solve problems involving the volume of cylinders, cones, and spheres;

(B) use previous knowledge of surface area to make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders;

(C) use the Pythagorean Theorem and its converse to solve problems; and

(D) determine the distance between two points on a coordinate plane using the Pythagorean Theorem.

(8) Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to:

(A) write one-variable equations or inequalities with variables on both sides that represent problems using rational number coefficients and constants;

(B) write a corresponding real-world problem when given a one-variable equation or inequality with variables on both sides of the equal sign using rational number coefficients and constants;

(C) model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants; and

(D) use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

(9) Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to develop foundational concepts of simultaneous linear equations. The student is expected to identify and verify the values of \( x \) and \( y \) that simultaneously satisfy two linear equations in the form \( y = mx + b \) from the intersections of the graphed equations.

(10) Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane;

(B) differentiate between transformations that preserve congruence and those that do not;
(C) explain the effect of translations, reflections over the $x$- or $y$-axis, and rotations limited to $90^\circ$, $180^\circ$, $270^\circ$, and $360^\circ$ as applied to two-dimensional shapes on a coordinate plane using an algebraic representation; and

(D) model the effect on linear and area measurements of dilated two-dimensional shapes.

(11) Measurement and data. The student applies mathematical process standards to use statistical procedures to describe data. The student is expected to:

(A) construct a scatterplot and describe the observed data to address questions of association such as linear, non-linear, and no association between bivariate data;

(B) determine the mean absolute deviation and use this quantity as a measure of the average distance data are from the mean using a data set of no more than 10 data points; and

(C) simulate generating random samples of the same size from a population with known characteristics to develop the notion of a random sample being representative of the population from which it was selected.

(12) Personal financial literacy. The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor. The student is expected to:

(A) solve real-world problems comparing how interest rate and loan length affect the cost of credit;

(B) calculate the total cost of repaying a loan, including credit cards and easy access loans, under various rates of interest and over different periods using an online calculator;

(C) explain how small amounts of money invested regularly, including money saved for college and retirement, grow over time;

(D) calculate and compare simple interest and compound interest earnings;

(E) identify and explain the advantages and disadvantages of different payment methods;

(F) analyze situations to determine if they represent financially responsible decisions and identify the benefits of financial responsibility and the costs of financial irresponsibility; and

(G) estimate the cost of a two-year and four-year college education, including family contribution, and devise a periodic savings plan for accumulating the money needed to contribute to the total cost of attendance for at least the first year of college.

Source: The provisions of this §111.28 adopted to be effective September 10, 2012, 37 TexReg 7109.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter C. High School

Statutory Authority: The provisions of this Subchapter C issued under the Texas Education Code, §§7.102(c)(4), 28.002, 28.008, and 28.025, unless otherwise noted.

(a) The provisions of §§111.39-111.45 of this subchapter shall be implemented by school districts.

(b) No later than June 30, 2015, the commissioner of education shall determine whether instructional materials funding has been made available to Texas public schools for materials that cover the essential knowledge and skills for mathematics as adopted in §§111.39-111.45 of this subchapter.

(c) If the commissioner makes the determination that instructional materials funding has been made available under subsection (b) of this section, §§111.39-111.45 of this subchapter shall be implemented beginning with the 2015-2016 school year and apply to the 2015-2016 and subsequent school years.

(d) If the commissioner does not make the determination that instructional materials funding has been made available under subsection (b) of this section, the commissioner shall determine no later than June 30 of each subsequent school year whether instructional materials funding has been made available. If the commissioner determines that instructional materials funding has been made available, the commissioner shall notify the State Board of Education and school districts that §§111.39-111.45 of this subchapter shall be implemented for the following school year.

(e) Sections 111.31-111.37 of this subchapter shall be superseded by the implementation of §§111.38-111.45 under this section.

Source: The provisions of this §111.38 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.39. Algebra I, Adopted 2012 (One Credit).

(a) General requirements. Students shall be awarded one credit for successful completion of this course. This course is recommended for students in Grade 8 or 9. Prerequisite: Mathematics, Grade 8 or its equivalent.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Algebra I, students will build on the knowledge and skills for mathematics in Grades 6-8, which provide a foundation in linear relationships, number and operations, and proportionality. Students will study linear, quadratic, and exponential functions and their related transformations, equations, and associated solutions. Students will connect functions and their associated solutions in both mathematical and real-world situations. Students will use technology to collect and explore data and analyze statistical relationships. In addition, students will study polynomials of degree one and two, radical expressions, sequences, and laws of exponents. Students will generate and solve linear systems with two equations and two variables and will create new functions through transformations.
§111.C. High School

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities;

(B) write linear equations in two variables in various forms, including \( y = mx + b \), \( Ax + By = C \), and \( y - y_1 = m(x - x_1) \), given one point and the slope and given two points;

(C) write linear equations in two variables given a table of values, a graph, and a verbal description;

(D) write and solve equations involving direct variation;

(E) write the equation of a line that contains a given point and is parallel to a given line;

(F) write the equation of a line that contains a given point and is perpendicular to a given line;

(G) write an equation of a line that is parallel or perpendicular to the X or Y axis and determine whether the slope of the line is zero or undefined;

(H) write linear inequalities in two variables given a table of values, a graph, and a verbal description; and

(I) write systems of two linear equations given a table of values, a graph, and a verbal description.

(3) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:
(A) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including \( y = mx + b \), \( Ax + By = C \), and \( y - y_1 = \frac{m}{x - x_1} \);
(B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems;
(C) graph linear functions on the coordinate plane and identify key features, including \( x \)-intercept, \( y \)-intercept, zeros, and slope, in mathematical and real-world problems;
(D) graph the solution set of linear inequalities in two variables on the coordinate plane;
(E) determine the effects on the graph of the parent function \( f(x) = x \) when \( f(x) \) is replaced by \( af(x) \), \( f(x) + d \), \( f(x - c) \), \( f(bx) \) for specific values of \( a \), \( b \), \( c \), and \( d \);
(F) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist;
(G) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems; and
(H) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

(4) Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:

(A) calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association;
(B) compare and contrast association and causation in real-world problems; and
(C) write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(5) Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(A) solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides;
(B) solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides; and
(C) solve systems of two linear equations with two variables for mathematical and real-world problems.

(6) Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:

(A) determine the domain and range of quadratic functions and represent the domain and range using inequalities;
(B) write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form \( f(x) = a(x - h)^2 + k \), and rewrite the equation from vertex form to standard form \( f(x) = ax^2 + bx + c \); and
(C) write quadratic functions when given real solutions and graphs of their related equations.

(7) Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:
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(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry;

(B) describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions; and

(C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x), f(x) + d, f(x - c), f(bx)$ for specific values of $a, b, c,$ and $d$.

(8) Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula; and

(B) write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(9) Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities;

(B) interpret the meaning of the values of $a$ and $b$ in exponential functions of the form $f(x) = ab^x$ in real-world problems;

(C) write exponential functions in the form $f(x) = ab^x$ (where $b$ is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay;

(D) graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems; and

(E) write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

(10) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:

(A) add and subtract polynomials of degree one and degree two;

(B) multiply polynomials of degree one and degree two;

(C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend;

(D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property;

(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two; and

(F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.
(11) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:

(A) simplify numerical radical expressions involving square roots; and
(B) simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

(12) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function;
(B) evaluate functions, expressed in function notation, given one or more elements in their domains;
(C) identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes;
(D) write a formula for the $a_n$th term of arithmetic and geometric sequences, given the value of several of their terms; and
(E) solve mathematic and scientific formulas, and other literal equations, for a specified variable.

Source: The provisions of this §111.39 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.40. Algebra II, Adopted 2012 (One-Half to One Credit).

(a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Algebra II, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. Students will broaden their knowledge of quadratic functions, exponential
functions, and systems of equations. Students will study logarithmic, square root, cubic, cube root, absolute value, rational functions, and their related equations. Students will connect functions to their inverses and associated equations and solutions in both mathematical and real-world situations. In addition, students will extend their knowledge of data analysis and numeric and algebraic methods.

(4) Statements that contain the word “including” reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;
(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
(E) create and use representations to organize, record, and communicate mathematical ideas;
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
(G) display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Attributes of functions and their inverses. The student applies mathematical processes to understand that functions have distinct key attributes and understand the relationship between a function and its inverse. The student is expected to:

(A) graph the functions \( f(x) = \sqrt{x}, f(x) = \frac{1}{x}, f(x) = x^3, f(x) = \sqrt[3]{x}, f(x) = b^x, f(x) = |x|, \) and \( f(x) = \log_b(x) \) where \( b \) is 2, 10, and \( e \), and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval;
(B) graph and write the inverse of a function using notation such as \( f^{-1}(x) \);
(C) describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range; and
(D) use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.

(3) Systems of equations and inequalities. The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve, and analyze reasonableness of solutions. The student is expected to:

(A) formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic;
(B) solve systems of three linear equations in three variables by using Gaussian elimination, technology with matrices, and substitution;
(C) solve, algebraically, systems of two equations in two variables consisting of a linear equation and a quadratic equation;
(D) determine the reasonableness of solutions to systems of a linear equation and a quadratic equation in two variables;

(E) formulate systems of at least two linear inequalities in two variables;

(F) solve systems of two or more linear inequalities in two variables; and

(G) determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.

(4) Quadratic and square root functions, equations, and inequalities. The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

(A) write the quadratic function given three specified points in the plane;

(B) write the equation of a parabola using given attributes, including vertex, focus, directrix, axis of symmetry, and direction of opening;

(C) determine the effect on the graph of \( f(x) = \sqrt{x} \) when \( f(x) \) is replaced by \( af(x) \), \( f(x) + d \), \( f(bx) \), and \( f(x - c) \) for specific positive and negative values of \( a \), \( b \), \( c \), and \( d \);

(D) transform a quadratic function \( f(x) = ax^2 + bx + c \) to the form \( f(x) = a(x - h)^2 + k \) to identify the different attributes of \( f(x) \);

(E) formulate quadratic and square root equations using technology given a table of data;

(F) solve quadratic and square root equations;

(G) identify extraneous solutions of square root equations; and

(H) solve quadratic inequalities.

(5) Exponential and logarithmic functions and equations. The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:

(A) determine the effects on the key attributes on the graphs of \( f(x) = b^x \) and \( f(x) = \log_b (x) \) where \( b = 2 \), \( 10 \), and \( e \) when \( f(x) \) is replaced by \( af(x) \), \( f(x) + d \), \( f(bx) \), and \( f(x - c) \) for specific positive and negative real values of \( a \), \( c \), and \( d \);

(B) formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation;

(C) rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations;

(D) solve exponential equations of the form \( y = ab^x \) where \( a \) is a nonzero real number and \( b \) is greater than zero and not equal to one and single logarithmic equations having real solutions; and

(E) determine the reasonableness of a solution to a logarithmic equation.

(6) Cubic, cube root, absolute value and rational functions, equations, and inequalities. The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

(A) analyze the effect on the graphs of \( f(x) = x^3 \) and \( f(x) = \sqrt[3]{x} \) when \( f(x) \) is replaced by \( af(x) \), \( f(bx) \), \( f(x - c) \), and \( f(x) + d \) for specific positive and negative real values of \( a \), \( b \), \( c \), and \( d \);

(B) solve cube root equations that have real roots;

(C) analyze the effect on the graphs of \( f(x) = |x| \) when \( f(x) \) is replaced by \( af(x) \), \( f(bx) \), \( f(x - c) \), and \( f(x) + d \) for specific positive and negative real values of \( a \), \( b \), \( c \), and \( d \).
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(D) formulate absolute value linear equations;

(E) solve absolute value linear equations;

(F) solve absolute value linear inequalities;

(G) analyze the effect on the graphs of \( f(x) = \frac{1}{x} \) when \( f(x) \) is replaced by \( af(x) \), \( f(bx) \), \( f(x-c) \), and \( f(x) + d \) for specific positive and negative real values of \( a \), \( b \), \( c \), and \( d \);

(H) formulate rational equations that model real-world situations;

(I) solve rational equations that have real solutions;

(J) determine the reasonableness of a solution to a rational equation;

(K) determine the asymptotic restrictions on the domain of a rational function and represent domain and range using interval notation, inequalities, and set notation; and

(L) formulate and solve equations involving inverse variation.

(7) Number and algebraic methods. The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:

(A) add, subtract, and multiply complex numbers;

(B) add, subtract, and multiply polynomials;

(C) determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two;

(D) determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods;

(E) determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping;

(F) determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and of degree two;

(G) rewrite radical expressions that contain variables to equivalent forms;

(H) solve equations involving rational exponents; and

(I) write the domain and range of a function in interval notation, inequalities, and set notation.

(8) Data. The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions. The student is expected to:

(A) analyze data to select the appropriate model from among linear, quadratic, and exponential models;

(B) use regression methods available through technology to write a linear function, a quadratic function, and an exponential function from a given set of data; and

(C) predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.

Source: The provisions of this §111.40 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.41. Geometry, Adopted 2012 (One Credit).

(a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.

(b) Introduction.
The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

In Geometry, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I to strengthen their mathematical reasoning skills in geometric contexts. Within the course, students will begin to focus on more precise terminology, symbolic representations, and the development of proofs. Students will explore concepts covering coordinate and transformational geometry; logical argument and constructions; proof and congruence; similarity, proof, and trigonometry; two- and three-dimensional figures; circles; and probability. Students will connect previous knowledge from Algebra I to Geometry through the coordinate and transformational geometry strand. In the logical arguments and constructions strand, students are expected to create formal constructions using a straight edge and compass. Though this course is primarily Euclidean geometry, students should complete the course with an understanding that non-Euclidean geometries exist. In proof and congruence, students will use deductive reasoning to justify, prove and apply theorems about geometric figures. Throughout the standards, the term "prove" means a formal proof to be shown in a paragraph, a flow chart, or two-column formats. Proportionality is the unifying component of the similarity, proof, and trigonometry strand. Students will use their proportional reasoning skills to prove and apply theorems and solve problems in this strand. The two- and three-dimensional figure strand focuses on the application of formulas in multi-step situations since students have developed background knowledge in two- and three-dimensional figures. Using patterns to identify geometric properties, students will apply theorems about circles to determine relationships between special segments and angles in circles. Due to the emphasis of probability and statistics in the college and career readiness standards, standards dealing with probability have been added to the geometry curriculum to ensure students have proper exposure to these topics before pursuing their post-secondary education.

These standards are meant to provide clarity and specificity in regards to the content covered in the high school geometry course. These standards are not meant to limit the methodologies used to convey this knowledge to students. Though the standards are written in a particular order, they are not necessarily meant to be taught in the given order. In the standards, the phrase "to solve problems" includes both contextual and non-contextual problems unless specifically stated.

Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

Knowledge and skills.
§111.C. Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

(A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint;

(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines; and

(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point.

(3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(A) describe and perform transformations of figures in a plane using coordinate notation;

(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane;

(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane; and

(D) identify and distinguish between reflectional and rotational symmetry in a plane figure.

(4) Logical argument and constructions. The student uses the process skills with deductive reasoning to understand geometric relationships. The student is expected to:

(A) distinguish between undefined terms, definitions, postulates, conjectures, and theorems;

(B) identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse;

(C) verify that a conjecture is false using a counterexample; and
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(D) compare geometric relationships between Euclidean and spherical geometries, including parallel lines and the sum of the angles in a triangle.

(5) Logical argument and constructions. The student uses constructions to validate conjectures about geometric figures. The student is expected to:

(A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools;

(B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge;

(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships; and

(D) verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems.

(6) Proof and congruence. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

(A) verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems;

(B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions;

(C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles;

(D) verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, midsegments, and medians, and apply these relationships to solve problems; and

(E) prove a quadrilateral is a parallelogram, rectangle, square, or rhombus using opposite sides, opposite angles, or diagonals and apply these relationships to solve problems.

(7) Similarity, proof, and trigonometry. The student uses the process skills in applying similarity to solve problems. The student is expected to:

(A) apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles; and

(B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.

(8) Similarity, proof, and trigonometry. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

(A) prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems; and

(B) identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems.
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(9) Similarity, proof, and trigonometry. The student uses the process skills to understand and apply relationships in right triangles. The student is expected to:

(A) determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems; and

(B) apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean theorem, including Pythagorean triples, to solve problems.

(10) Two-dimensional and three-dimensional figures. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:

(A) identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones, and spheres and identify three-dimensional objects generated by rotations of two-dimensional shapes; and

(B) determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.

(11) Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:

(A) apply the formula for the area of regular polygons to solve problems using appropriate units of measure;

(B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure;

(C) apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure; and

(D) apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure.

(12) Circles. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to:

(A) apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems;

(B) apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems;

(C) apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems;

(D) describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle; and

(E) show that the equation of a circle with center at the origin and radius \( r \) is \( x^2 + y^2 = r^2 \) and determine the equation for the graph of a circle with radius \( r \) and center \((h, k)\), \((x - h)^2 + (y - k)^2 = r^2\).

(13) Probability. The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to:

(A) develop strategies to use permutations and combinations to solve contextual problems;

(B) determine probabilities based on area to solve contextual problems;
(C) identify whether two events are independent and compute the probability of the two events occurring together with or without replacement;

(D) apply conditional probability in contextual problems; and

(E) apply independence in contextual problems.

Source: The provisions of this §111.41 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.42. Precalculus, Adopted 2012 (One-Half to One Credit).

(a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Algebra I, Geometry, and Algebra II.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) Precalculus is the preparation for calculus. The course approaches topics from a function point of view, where appropriate, and is designed to strengthen and enhance conceptual understanding and mathematical reasoning used when modeling and solving mathematical and real-world problems. Students systematically work with functions and their multiple representations. The study of Precalculus deepens students' mathematical understanding and fluency with algebra and trigonometry and extends their ability to make connections and apply concepts and procedures at higher levels. Students investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use technology to build understanding, make connections between representations, and provide support in solving problems.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;
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(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Functions. The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

(A) use the composition of two functions to model and solve real-world problems;

(B) demonstrate that function composition is not always commutative;

(C) represent a given function as a composite function of two or more functions;

(D) describe symmetry of graphs of even and odd functions;

(E) determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;

(F) graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;

(G) graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including \( af(x) \), \( f(x) + d \), \( f(x - c) \), \( f(bx) \) for specific values of \( a \), \( b \), \( c \), and \( d \), in mathematical and real-world problems;

(H) graph arcsin \( x \) and arccos \( x \) and describe the limitations on the domain;

(I) determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;

(J) analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems;

(K) analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes;

(L) determine various types of discontinuities in the interval \((-\infty, \infty)\) as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities;

(M) describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities;

(N) analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems;
(O) develop and use a sinusoidal function that models a situation in mathematical and real-world problems; and

(P) determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.

(3) Relations and geometric reasoning. The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

(A) graph a set of parametric equations;

(B) convert parametric equations into rectangular relations and convert rectangular relations into parametric equations;

(C) use parametric equations to model and solve mathematical and real-world problems;

(D) graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates;

(E) graph polar equations by plotting points and using technology;

(F) determine the conic section formed when a plane intersects a double-napped cone;

(G) make connections between the locus definition of conic sections and their equations in rectangular coordinates;

(H) use the characteristics of an ellipse to write the equation of an ellipse with center \((h, k)\); and

(I) use the characteristics of a hyperbola to write the equation of a hyperbola with center \((h, k)\).

(4) Number and measure. The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

(A) determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems;

(B) describe the relationship between degree and radian measure on the unit circle;

(C) represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position;

(D) represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity;

(E) determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems;

(F) use trigonometry in mathematical and real-world problems, including directional bearing;

(G) use the Law of Sines in mathematical and real-world problems;

(H) use the Law of Cosines in mathematical and real-world problems;

(I) use vectors to model situations involving magnitude and direction;

(J) represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically; and

(K) apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.
§111.C. Algebraic reasoning. The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

(A) evaluate finite sums and geometric series, when possible, written in sigma notation;
(B) represent arithmetic sequences and geometric sequences using recursive formulas;
(C) calculate the $n^{th}$ term and the $n^{th}$ partial sum of an arithmetic series in mathematical and real-world problems;
(D) represent arithmetic series and geometric series using sigma notation;
(E) calculate the $n^{th}$ term of a geometric series, the $n^{th}$ partial sum of a geometric series, and sum of an infinite geometric series when it exists;
(F) apply the Binomial Theorem for the expansion of $(a + b)^n$ in powers of $a$ and $b$ for a positive integer $n$, where $a$ and $b$ are any numbers;
(G) use the properties of logarithms to evaluate or transform logarithmic expressions;
(H) generate and solve logarithmic equations in mathematical and real-world problems;
(I) generate and solve exponential equations in mathematical and real-world problems;
(J) solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems;
(K) solve polynomial inequalities with real coefficients by applying a variety of techniques and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems;
(L) solve rational inequalities with real coefficients by applying a variety of techniques and write the solution set of the rational inequality in interval notation in mathematical and real-world problems;
(M) use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions; and
(N) generate and solve trigonometric equations in mathematical and real-world problems.

Source: The provisions of this §111.42 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.43. Mathematical Models with Applications, Adopted 2012 (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I. This course must be taken before receiving credit for Algebra II.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and
evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) Mathematical Models with Applications is designed to build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. This mathematics course provides a path for students to succeed in Algebra II and prepares them for various post-secondary choices. Students learn to apply mathematics through experiences in personal finance, science, engineering, fine arts, and social sciences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, model information, solve problems, and communicate solutions. Students will select from tools such as physical objects; manipulatives; technology, including graphing calculators, data collection devices, and computers; and paper and pencil and from methods such as algebraic techniques, geometric reasoning, patterns, and mental math to solve problems.

(4) In Mathematical Models with Applications, students will use a mathematical modeling cycle to analyze problems, understand problems better, and improve decisions. A basic mathematical modeling cycle is summarized in this paragraph. The student will:

(A) represent:

(i) identify the variables in the problem and select those that represent essential features; and

(ii) formulate a model by creating and selecting from representations such as geometric, graphical, tabular, algebraic, or statistical that describe the relationships between the variables;

(B) compute: analyze and perform operations on the relationships between the variables to draw conclusions;

(C) interpret: interpret the results of the mathematics in terms of the original problem;

(D) revise: confirm the conclusions by comparing the conclusions with the problem and revising as necessary; and

(E) report: report on the conclusions and the reasoning behind the conclusions.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
(E) create and use representations to organize, record, and communicate mathematical ideas;
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Mathematical modeling in personal finance. The student uses mathematical processes with graphical and numerical techniques to study patterns and analyze data related to personal finance. The student is expected to:
(A) use rates and linear functions to solve problems involving personal finance and budgeting, including compensations and deductions;
(B) solve problems involving personal taxes; and
(C) analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees.

(3) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. The student is expected to:
(A) use formulas to generate tables to display series of payments for loan amortizations resulting from financed purchases;
(B) analyze personal credit options in retail purchasing and compare relative advantages and disadvantages of each option;
(C) use technology to create amortization models to investigate home financing and compare buying a home to renting a home; and
(D) use technology to create amortization models to investigate automobile financing and compare buying a vehicle to leasing a vehicle.

(4) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:
(A) analyze and compare coverage options and rates in insurance;
(B) investigate and compare investment options, including stocks, bonds, annuities, certificates of deposit, and retirement plans; and
(C) analyze types of savings options involving simple and compound interest and compare relative advantages of these options.

(5) Mathematical modeling in science and engineering. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. The student is expected to:
(A) use proportionality and inverse variation to describe physical laws such as Hook's Law, Newton's Second Law of Motion, and Boyle's Law;
(B) use exponential models available through technology to model growth and decay in areas, including radioactive decay; and
(C) use quadratic functions to model motion.

(6) Mathematical modeling in science and engineering. The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. The student is expected to:
(A) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in architecture;
(B) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields;

(C) use the Pythagorean Theorem and special right-triangle relationships to calculate distances; and

(D) use trigonometric ratios to calculate distances and angle measures as applied to fields.

(7) Mathematical modeling in fine arts. The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. The student is expected to:

(A) use trigonometric ratios and functions available through technology to model periodic behavior in art and music;

(B) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography;

(C) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music; and

(D) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields.

(8) Mathematical modeling in social sciences. The student applies mathematical processes to determine the number of elements in a finite sample space and compute the probability of an event. The student is expected to:

(A) determine the number of ways an event may occur using combinations, permutations, and the Fundamental Counting Principle;

(B) compare theoretical to empirical probability; and

(C) use experiments to determine the reasonableness of a theoretical model such as binomial or geometric.

(9) Mathematical modeling in social sciences. The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. The student is expected to:

(A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, dot plots, stem-and-leaf plots, and box and whisker plots, to draw conclusions from the data and determine the strengths and weaknesses of conclusions;

(B) analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions;

(C) distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies;

(D) use data from a sample to estimate population mean or population proportion;

(E) analyze marketing claims based on graphs and statistics from electronic and print media and justify the validity of stated or implied conclusions; and

(F) use regression methods available through technology to model linear and exponential functions, interpret correlations, and make predictions.

(10) Mathematical modeling in social sciences. The student applies mathematical processes to design a study and use graphical, numerical, and analytical techniques to communicate the results of the study. The student is expected to:
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(A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions; and

(B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation.

Source: The provisions of this §111.43 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.44. Advanced Quantitative Reasoning, Adopted 2012 (One-Half to One Credit).

(a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Advanced Quantitative Reasoning, students will develop and apply skills necessary for college, careers, and life. Course content consists primarily of applications of high school mathematics concepts to prepare students to become well-educated and highly informed 21st century citizens. Students will develop and apply reasoning, planning, and communication to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection, and modeling with algebra, geometry, trigonometry, and discrete mathematics.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Numeric reasoning. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations. The student is expected to:

(A) use precision and accuracy in real-life situations related to measurement and significant figures;

(B) apply and analyze published ratings, weighted averages, and indices to make informed decisions;

(C) solve problems involving quantities that are not easily measured using proportionality;

(D) solve geometric problems involving indirect measurement, including similar triangles, the Pythagorean Theorem, Law of Sines, Law of Cosines, and the use of dynamic geometry software;

(E) solve problems involving large quantities using combinatorics;

(F) use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems, including geometric transformations;

(G) analyze various voting and selection processes to compare results in given situations; and

(H) select and apply an algorithm of interest to solve real-life problems such as problems using recursion or iteration involving population growth or decline, fractals, and compound interest; the validity in recorded and transmitted data using checksums and hashing; sports rankings, weighted class rankings, and search engine rankings; and problems involving scheduling or routing situations using vertex-edge graphs, critical paths, Euler paths, and minimal spanning trees and communicate to peers the application of the algorithm in precise mathematical and nontechnical language.

(3) Algebraic reasoning (expressions, equations, and generalized relationships). The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems. The student is expected to:

(A) collect numerical bivariate data to create a scatterplot, select a function to model the data, justify the model selection, and use the model to interpret results and make predictions;

(B) describe the degree to which uncorrelated variables may or may not be related and analyze situations where correlated variables do or do not indicate a cause-and-effect relationship;

(C) determine or analyze an appropriate growth or decay model for problem situations, including linear, exponential, and logistic functions;
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(D) determine or analyze an appropriate cyclical model for problem situations that can be modeled with periodic functions;

(E) determine or analyze an appropriate piecewise model for problem situations;

(F) create, represent, and analyze mathematical models for various types of income calculations to determine the best option for a given situation;

(G) create, represent, and analyze mathematical models for expenditures, including those involving credit, to determine the best option for a given situation; and

(H) create, represent, and analyze mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best option for a given situation.

(4) Probabilistic and statistical reasoning. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to:

(A) use a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results;

(B) use the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), in mathematical and real-world problems;

(C) calculate conditional probabilities and probabilities of compound events using tree diagrams, Venn diagrams, area models, and formulas;

(D) interpret conditional probabilities and probabilities of compound events by analyzing representations to make decisions in problem situations;

(E) use probabilities to make and justify decisions about risks in everyday life;

(F) calculate expected value to analyze mathematical fairness, payoff, and risk;

(G) determine the validity of logical arguments that include compound conditional statements by constructing truth tables;

(H) identify limitations and lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form;

(I) interpret and compare statistical results using appropriate technology given a margin of error;

(J) identify potential misuses of statistics to justify particular conclusions, including assertions of a cause-and-effect relationship rather than an association, and missteps or fallacies in logical reasoning;

(K) describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics and other results appearing in a study, including reports published in the media;

(L) determine the need for and purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions;

(M) identify the population of interest for a statistical investigation, select an appropriate sampling technique, and collect data;

(N) identify the variables to be used in a study;
(O) determine possible sources of statistical bias in a study and how bias may affect the validity of the results;

(P) create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features of the data;

(Q) analyze possible sources of data variability, including those that can be controlled and those that cannot be controlled;

(R) report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied;

(S) justify the design and the conclusion(s) of statistical studies, including the methods used; and

(T) communicate statistical results in oral and written formats using appropriate statistical and nontechnical language.

Source: The provisions of this §111.44 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.45. Independent Study in Mathematics, Adopted 2012 (One-Half to One Credit).

(a) General requirements.

(1) Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.

(2) Students may repeat this course with different course content for up to three credits.

(3) The requirements for each course must be approved by the local district before the course begins.

(4) If this course is being used to satisfy requirements for the Distinguished Achievement Program, student research/products must be presented before a panel of professionals or approved by the student's mentor.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
§111.C. In Independent Study in Mathematics, students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics such as theory of equations, number theory, non-Euclidean geometry, linear algebra, advanced survey of mathematics, or history of mathematics.

(4) Statements that contain the word “including” reference content that must be mastered, while those containing the phrase “such as” are intended as possible illustrative examples.

(c) Knowledge and skills: mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(1) apply mathematics to problems arising in everyday life, society, and the workplace;

(2) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(3) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(4) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(5) create and use representations to organize, record, and communicate mathematical ideas;

(6) analyze mathematical relationships to connect and communicate mathematical ideas; and

(7) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Source: The provisions of this §111.45 adopted to be effective September 10, 2012, 37 TexReg 7109.
Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter D. Other High School Mathematics Courses

Statutory Authority: The provisions of this Subchapter D issued under the Texas Education Code, §§7.102(c)(4), 28.002, and 28.025, unless otherwise noted.

§111.51. Implementation of Texas Essential Knowledge and Skills for Mathematics, Other High School Mathematics Courses.

The provisions of this subchapter shall be implemented by school districts.

Source: The provisions of this §111.51 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective September 10, 2012, 37 TexReg 7109.

§111.52. Independent Study in Mathematics (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of Independent Study in Mathematics. Required prerequisites: Algebra II, Geometry. Students may repeat this course with different course content for up to three credits.

(b) Content requirements. Students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics, such as theory of equations, number theory, non-Euclidean geometry, advanced survey of mathematics, or history of mathematics. The requirements for each course must be approved by the local district before the course begins.

(c) If this course is being used to satisfy requirements for the Distinguished Achievement Program, student research/products must be presented before a panel of professionals or approved by the student's mentor.

Source: The provisions of this §111.52 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective July 12, 2010, 35 TexReg 6069.

§111.53. Advanced Placement (AP) Statistics (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.

(b) Content requirements. Content requirements for Advanced Placement (AP) Statistics are prescribed in the College Board Publication Advanced Placement Course Description: Statistics, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.53 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.54. Advanced Placement (AP) Calculus AB (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.

(b) Content requirements. Content requirements for Advanced Placement (AP) Calculus AB are prescribed in the College Board Publication Advanced Placement Course Description Mathematics: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.54 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.55. Advanced Placement (AP) Calculus BC (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.
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(b) Content requirements. Content requirements for Advanced Placement (AP) Calculus BC are prescribed in the College Board Publication Advanced Placement Course Description: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.55 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.56. IB Mathematical Studies Standard Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematical Studies Standard Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: Algebra II, Geometry.

(b) Content requirements. Content requirements for IB Mathematical Studies Standard Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.56 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective October 21, 2007, 32 TexReg 7262.

§111.57. IB Mathematics Standard Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematics Standard Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: Algebra II, Geometry.

(b) Content requirements. Content requirements for IB Mathematics Standard Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.57 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective October 21, 2007, 32 TexReg 7262.

§111.58. IB Mathematics Higher Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematics Higher Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematical Studies Standard Level or IB Mathematics Standard Level.

(b) Content requirements. Content requirements for IB Mathematics Higher Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.58 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective October 21, 2007, 32 TexReg 7262.

§111.59. IB Further Mathematics Higher Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Further Mathematics Higher Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematics Higher Level.

(b) Content requirements. Content requirements for IB Further Mathematics Higher Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.
Other High School Mathematics Courses §111.D.

Source: The provisions of this §111.59 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective October 21, 2007, 32 TexReg 7262; amended to be effective September 10, 2012, 37 TexReg 7109; amended to be effective September 10, 2012, 37 TexReg 7109.

§111.60. Concurrent Enrollment in College Courses.

(a) General requirements. Students shall be awarded at least one-half credit for each semester of successful completion of a college course in which the student is concurrently enrolled while in high school.

(b) Content requirements. In order for students to receive state graduation credit for concurrent enrollment courses, content requirements must meet or exceed the essential knowledge and skills in a given course.

Source: The provisions of this §111.60 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective July 12, 2010, 35 TexReg 6069.

§111.61. Other Courses for Which Students May Receive Mathematics Credit.

(a) Mathematical Applications in Agriculture, Food, and Natural Resources. Students on the minimum high school program or recommended high school program shall be awarded one credit in mathematics for successful completion of this course as described in §130.10 of this title (relating to Mathematical Applications in Agriculture, Food, and Natural Resources (One Credit)) and in accordance with the graduation requirements in Chapter 74 of this title (relating to Curriculum Requirements). Recommended prerequisite: a minimum of one credit from the courses in the Agriculture, Food, and Natural Resources cluster.

(b) Engineering Mathematics. Students shall be awarded one credit in mathematics for successful completion of this course as described in §130.367 of this title (relating to Engineering Mathematics (One Credit)). Prerequisite: Algebra II. This course is recommended for students in Grades 11 and 12.

(c) Statistics and Risk Management. Students shall be awarded one credit in mathematics for successful completion of this course as described in §130.169 of this title (relating to Statistics and Risk Management (One Credit)). Recommended prerequisites: Accounting I and Algebra II. This course is recommended for students in Grades 11 and 12.

(d) Advanced Placement (AP) Computer Science A. Students may be awarded one mathematics credit for successful completion of this course. Content requirements for Advanced Placement (AP) Computer Science A are prescribed in the College Board Publication Advanced Placement Course Description: Computer Science A, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.61 adopted to be effective July 12, 2010, 35 TexReg 6069.
Texas Administrative Code, Title 19, Part 2, Chapter 74, Curriculum Requirements, Subchapter A

Statutory Authority: The provisions of this Subchapter A issued under the Texas Education Code, §§7.102, 28.002, 28.023, 28.025, 28.054, and 38.003, unless otherwise noted.

Rule §74.4. English Language Proficiency Standards.

[Note that we are here reprinting only §74.4, the ELPS, and not the preceding §§74.1, 74.2, and 74.3]

About the finding aids we have added to the English Language Proficiency Standards
We have made one change to the formatting of the English Language Proficiency Standards as they appear online in the Texas Administrative Code. We have added running footers on each page that specify “Chapter 74. Rule §74.4 English Language Proficiency Standards.”
§74.4. English Language Proficiency Standards.

(a) **Introduction.**

(1) The English language proficiency standards in this section outline English language proficiency level descriptors and student expectations for English language learners (ELLs). School districts shall implement this section as an integral part of each subject in the required curriculum. The English language proficiency standards are to be published along with the Texas Essential Knowledge and Skills (TEKS) for each subject in the required curriculum.

(2) In order for ELLs to be successful, they must acquire both social and academic language proficiency in English. Social language proficiency in English consists of the English needed for daily social interactions. Academic language proficiency consists of the English needed to think critically, understand and learn new concepts, process complex academic material, and interact and communicate in English academic settings.

(3) Classroom instruction that effectively integrates second language acquisition with quality content area instruction ensures that ELLs acquire social and academic language proficiency in English, learn the knowledge and skills in the TEKS, and reach their full academic potential.

(4) Effective instruction in second language acquisition involves giving ELLs opportunities to listen, speak, read, and write at their current levels of English development while gradually increasing the linguistic complexity of the English they read and hear, and are expected to speak and write.

(5) The cross-curricular second language acquisition skills in subsection (c) of this section apply to ELLs in Kindergarten–Grade 12.

(6) The English language proficiency levels of beginning, intermediate, advanced, and advanced high are not grade-specific. ELLs may exhibit different proficiency levels within the language domains of listening, speaking, reading, and writing. The proficiency level descriptors outlined in subsection (d) of this section show the progression of second language acquisition from one proficiency level to the next and serve as a road map to help content area teachers instruct ELLs commensurate with students' linguistic needs.

(b) **School district responsibilities. In fulfilling the requirements of this section, school districts shall:**

(1) identify the student’s English language proficiency levels in the domains of listening, speaking, reading, and writing in accordance with the proficiency level descriptors for the beginning, intermediate, advanced, and advanced high levels delineated in subsection (d) of this section;

(2) provide instruction in the knowledge and skills of the foundation and enrichment curriculum in a manner that is linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s levels of English language proficiency to ensure that the student learns the knowledge and skills in the required curriculum;

(3) provide content-based instruction including the cross-curricular second language acquisition essential knowledge and skills in subsection (c) of this section in a manner that is linguistically accommodated to help the student acquire English language proficiency; and

(4) provide intensive and ongoing foundational second language acquisition instruction to ELLs in Grade 3 or higher who are at the beginning or intermediate level of English language proficiency in listening, speaking, reading, and/or writing as determined by the state’s English language proficiency assessment system. These ELLs require focused, targeted, and systematic second language acquisition instruction to provide them with the foundation of English language vocabulary, grammar, syntax, and English mechanics necessary to support content-based instruction and accelerated learning of English.

(c) **Cross-curricular second language acquisition essential knowledge and skills.**

(1) Cross-curricular second language acquisition/learning strategies. The ELL uses language learning strategies to develop an awareness of his or her own learning processes in all content areas. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. The student is expected to:
(A) use prior knowledge and experiences to understand meanings in English;

(B) monitor oral and written language production and employ self-corrective techniques or other resources;

(C) use strategic learning techniques such as concept mapping, drawing, memorizing, comparing, contrasting, and reviewing to acquire basic and grade-level vocabulary;

(D) speak using learning strategies such as requesting assistance, employing non-verbal cues, and using synonyms and circumlocution (conveying ideas by defining or describing when exact English words are not known);

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment;

(F) use accessible language and learn new and essential language in the process;

(G) demonstrate an increasing ability to distinguish between formal and informal English and an increasing knowledge of when to use each one commensurate with grade-level learning expectations; and

(H) develop and expand repertoire of learning strategies such as reasoning inductively or deductively, looking for patterns in language, and analyzing sayings and expressions commensurate with grade-level learning expectations.

(2) Cross-curricular second language acquisition/listening. The ELL listens to a variety of speakers including teachers, peers, and electronic media to gain an increasing level of comprehension of newly acquired language in all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in listening. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. The student is expected to:

(A) distinguish sounds and intonation patterns of English with increasing ease;

(B) recognize elements of the English sound system in newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters;

(C) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions;

(D) monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed;

(E) use visual, contextual, and linguistic support to enhance and confirm understanding of increasingly complex and elaborated spoken language;

(F) listen to and derive meaning from a variety of media such as audio tape, video, DVD, and CD ROM to build and reinforce concept and language attainment;

(G) understand the general meaning, main points, and important details of spoken language ranging from situations in which topics, language, and contexts are familiar to unfamiliar;

(H) understand implicit ideas and information in increasingly complex spoken language commensurate with grade-level learning expectations; and

(I) demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.
(3) Cross-curricular second language acquisition/speaking. The ELL speaks in a variety of modes for a variety of purposes with an awareness of different language registers (formal/informal) using vocabulary with increasing fluency and accuracy in language arts and all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in speaking. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. The student is expected to:

(A) practice producing sounds of newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters to pronounce English words in a manner that is increasingly comprehensible;

(B) expand and internalize initial English vocabulary by learning and using high-frequency English words necessary for identifying and describing people, places, and objects, by retelling simple stories and basic information represented or supported by pictures, and by learning and using routine language needed for classroom communication;

(C) speak using a variety of grammatical structures, sentence lengths, sentence types, and connecting words with increasing accuracy and ease as more English is acquired;

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency;

(E) share information in cooperative learning interactions;

(F) ask and give information ranging from using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments;

(G) express opinions, ideas, and feelings ranging from communicating single words and short phrases to participating in extended discussions on a variety of social and grade-appropriate academic topics;

(H) narrate, describe, and explain with increasing specificity and detail as more English is acquired;

(I) adapt spoken language appropriately for formal and informal purposes; and

(J) respond orally to information presented in a wide variety of print, electronic, audio, and visual media to build and reinforce concept and language attainment.

(4) Cross-curricular second language acquisition/reading. The ELL reads a variety of texts for a variety of purposes with an increasing level of comprehension in all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in reading. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. For Kindergarten and Grade 1, certain of these student expectations apply to text read aloud for students not yet at the stage of decoding written text. The student is expected to:

(A) learn relationships between sounds and letters of the English language and decode (sound out) words using a combination of skills such as recognizing sound-letter relationships and identifying cognates, affixes, roots, and base words;

(B) recognize directionality of English reading such as left to right and top to bottom;

(C) develop basic sight vocabulary, derive meaning of environmental print, and comprehend English vocabulary and language structures used routinely in written classroom materials;

(D) use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary and other prereading activities to enhance comprehension of written text;
(E) read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned;

(F) use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language;

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade level needs;

(H) read silently with increasing ease and comprehension for longer periods;

(I) demonstrate English comprehension and expand reading skills by employing basic reading skills such as demonstrating understanding of supporting ideas and details in text and graphic sources, summarizing text, and distinguishing main ideas from details commensurate with content area needs;

(J) demonstrate English comprehension and expand reading skills by employing inferential skills such as predicting, making connections between ideas, drawing inferences and conclusions from text and graphic sources, and finding supporting text evidence commensurate with content area needs; and

(K) demonstrate English comprehension and expand reading skills by employing analytical skills such as evaluating written information and performing critical analyses commensurate with content area and grade-level needs.

(5) Cross-curricular second language acquisition/writing. The ELL writes in a variety of forms with increasing accuracy to effectively address a specific purpose and audience in all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in writing. In order for the ELL to meet grade-level learning expectations across foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student’s level of English language proficiency. For Kindergarten and Grade 1, certain of these student expectations do not apply until the student has reached the stage of generating original written text using a standard writing system. The student is expected to:

(A) learn relationships between sounds and letters of the English language to represent sounds when writing in English;

(B) write using newly acquired basic vocabulary and content-based grade-level vocabulary;

(C) spell familiar English words with increasing accuracy, and employ English spelling patterns and rules with increasing accuracy as more English is acquired;

(D) edit writing for standard grammar and usage, including subject-verb agreement, pronoun agreement, and appropriate verb tenses commensurate with grade-level expectations as more English is acquired;

(E) employ increasingly complex grammatical structures in content area writing commensurate with grade-level expectations, such as:

   (i) using correct verbs, tenses, and pronouns/antecedents;

   (ii) using possessive case (apostrophe’s) correctly; and

   (iii) using negatives and contractions correctly;

(F) write using a variety of grade-appropriate sentence lengths, patterns, and connecting words to combine phrases, clauses, and sentences in increasingly accurate ways as more English is acquired; and

(G) narrate, describe, and explain with increasing specificity and detail to fulfill content area writing needs as more English is acquired.
(d) Proficiency level descriptors.

(1) Listening, Kindergarten-Grade 12. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in listening. The following proficiency level descriptors for listening are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction.

(A) Beginning. Beginning ELLs have little or no ability to understand spoken English in academic and social settings. These students:

(i) struggle to understand simple conversations and simple discussions even when the topics are familiar and the speaker uses linguistic supports such as visuals, slower speech and other verbal cues, and gestures;

(ii) struggle to identify and distinguish individual words and phrases during social and instructional interactions that have not been intentionally modified for ELLs; and

(iii) may not seek clarification in English when failing to comprehend the English they hear; frequently remain silent, watching others for cues.

(B) Intermediate. Intermediate ELLs have the ability to understand simple, high-frequency spoken English used in routine academic and social settings. These students:

(i) usually understand simple or routine directions, as well as short, simple conversations and short, simple discussions on familiar topics; when topics are unfamiliar, require extensive linguistic supports and adaptations such as visuals, slower speech and other verbal cues, simplified language, gestures, and preteaching to preview or build topic-related vocabulary;

(ii) often identify and distinguish key words and phrases necessary to understand the general meaning during social and basic instructional interactions that have not been intentionally modified for ELLs; and

(iii) have the ability to seek clarification in English when failing to comprehend the English they hear by requiring/requesting the speaker to repeat, slow down, or rephrase speech.

(C) Advanced. Advanced ELLs have the ability to understand, with second language acquisition support, grade-appropriate spoken English used in academic and social settings. These students:

(i) usually understand longer, more elaborated directions, conversations, and discussions on familiar and some unfamiliar topics, but sometimes need processing time and sometimes depend on visuals, verbal cues, and gestures to support understanding;

(ii) understand most main points, most important details, and some implicit information during social and basic instructional interactions that have not been intentionally modified for ELLs; and

(iii) occasionally require/request the speaker to repeat, slow down, or rephrase to clarify the meaning of the English they hear.

(D) Advanced high. Advanced high ELLs have the ability to understand, with minimal second language acquisition support, grade-appropriate spoken English used in academic and social settings. These students:

(i) understand longer, elaborated directions, conversations, and discussions on familiar and unfamiliar topics with occasional need for processing time and with little dependence on visuals, verbal cues, and gestures; some exceptions when complex academic or highly specialized language is used;
(ii) understand main points, important details, and implicit information at a level nearly comparable to native English-speaking peers during social and instructional interactions; and

(iii) rarely require/request the speaker to repeat, slow down, or rephrase to clarify the meaning of the English they hear.

(2) Speaking, Kindergarten–Grade 12. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in speaking. The following proficiency level descriptors for speaking are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction.

(A) Beginning. Beginning ELLs have little or no ability to speak English in academic and social settings. These students:

   (i) mainly speak using single words and short phrases consisting of recently practiced, memorized, or highly familiar material to get immediate needs met; may be hesitant to speak and often give up in their attempts to communicate;

   (ii) speak using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts;

   (iii) lack the knowledge of English grammar necessary to connect ideas and speak in sentences; can sometimes produce sentences using recently practiced, memorized, or highly familiar material;

   (iv) exhibit second language acquisition errors that may hinder overall communication, particularly when trying to convey information beyond memorized, practiced, or highly familiar material; and

   (v) typically use pronunciation that significantly inhibits communication.

(B) Intermediate. Intermediate ELLs have the ability to speak in a simple manner using English commonly heard in routine academic and social settings. These students:

   (i) are able to express simple, original messages, speak using sentences, and participate in short conversations and classroom interactions; may hesitate frequently and for long periods to think about how to communicate desired meaning;

   (ii) speak simply using basic vocabulary needed in everyday social interactions and routine academic contexts; rarely have vocabulary to speak in detail;

   (iii) exhibit an emerging awareness of English grammar and speak using mostly simple sentence structures and simple tenses; are most comfortable speaking in present tense;

   (iv) exhibit second language acquisition errors that may hinder overall communication when trying to use complex or less familiar English; and

   (v) use pronunciation that can usually be understood by people accustomed to interacting with ELLs.

(C) Advanced. Advanced ELLs have the ability to speak using grade-appropriate English, with second language acquisition support, in academic and social settings. These students:

   (i) are able to participate comfortably in most conversations and academic discussions on familiar topics, with some pauses to restate, repeat, or search for words and phrases to clarify meaning;

   (ii) discuss familiar academic topics using content-based terms and common abstract vocabulary; can usually speak in some detail on familiar topics;
(iii) have a grasp of basic grammar features, including a basic ability to narrate and describe in present, past, and future tenses; have an emerging ability to use complex sentences and complex grammar features;

(iv) make errors that interfere somewhat with communication when using complex grammar structures, long sentences, and less familiar words and expressions; and

(v) may mispronounce words, but use pronunciation that can usually be understood by people not accustomed to interacting with ELLs.

(D) Advanced high. Advanced high ELLs have the ability to speak using grade-appropriate English, with minimal second language acquisition support, in academic and social settings. These students:

(i) are able to participate in extended discussions on a variety of social and grade-appropriate academic topics with only occasional disruptions, hesitations, or pauses;

(ii) communicate effectively using abstract and content-based vocabulary during classroom instructional tasks, with some exceptions when low-frequency or academically demanding vocabulary is needed; use many of the same idioms and colloquialisms as their native English-speaking peers;

(iii) can use English grammar structures and complex sentences to narrate and describe at a level nearly comparable to native English-speaking peers;

(iv) make few second language acquisition errors that interfere with overall communication; and

(v) may mispronounce words, but rarely use pronunciation that interferes with overall communication.

(3) Reading, Kindergarten–Grade 1. ELLs in Kindergarten and Grade 1 may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in reading. The following proficiency level descriptors for reading are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction and should take into account developmental stages of emergent readers.

(A) Beginning. Beginning ELLs have little or no ability to use the English language to build foundational reading skills. These students:

(i) derive little or no meaning from grade-appropriate stories read aloud in English, unless the stories are:

(I) read in short “chunks;”

(II) controlled to include the little English they know such as language that is high frequency, concrete, and recently practiced; and

(III) accompanied by ample visual supports such as illustrations, gestures, pantomime, and objects and by linguistic supports such as careful enunciation and slower speech;

(ii) begin to recognize and understand environmental print in English such as signs, labeled items, names of peers, and logos; and

(iii) have difficulty decoding most grade-appropriate English text because they:

(I) understand the meaning of very few words in English; and

(II) struggle significantly with sounds in spoken English words and with sound-symbol relationships due to differences between their primary language and English.
(B) Intermediate. Intermediate ELLs have a limited ability to use the English language to build foundational reading skills. These students:

(i) demonstrate limited comprehension (key words and general meaning) of grade-appropriate stories read aloud in English, unless the stories include:

(I) predictable story lines;
(II) highly familiar topics;
(III) primarily high-frequency, concrete vocabulary;
(IV) short, simple sentences; and
(V) visual and linguistic supports;

(ii) regularly recognize and understand common environmental print in English such as signs, labeled items, names of peers, logos; and

(iii) have difficulty decoding grade-appropriate English text because they:

(I) understand the meaning of only those English words they hear frequently; and

(II) struggle with some sounds in English words and some sound-symbol relationships due to differences between their primary language and English.

(C) Advanced. Advanced ELLs have the ability to use the English language, with second language acquisition support, to build foundational reading skills. These students:

(i) demonstrate comprehension of most main points and most supporting ideas in grade-appropriate stories read aloud in English, although they may still depend on visual and linguistic supports to gain or confirm meaning;

(ii) recognize some basic English vocabulary and high-frequency words in isolated print; and

(iii) with second language acquisition support, are able to decode most grade-appropriate English text because they:

(I) understand the meaning of most grade-appropriate English words; and

(II) have little difficulty with English sounds and sound-symbol relationships that result from differences between their primary language and English.

(D) Advanced high. Advanced high ELLs have the ability to use the English language, with minimal second language acquisition support, to build foundational reading skills. These students:

(i) demonstrate, with minimal second language acquisition support and at a level nearly comparable to native English-speaking peers, comprehension of main points and supporting ideas (explicit and implicit) in grade-appropriate stories read aloud in English;

(ii) with some exceptions, recognize sight vocabulary and high-frequency words to a degree nearly comparable to that of native English-speaking peers; and

(iii) with minimal second language acquisition support, have an ability to decode and understand grade-appropriate English text at a level nearly comparable to native English-speaking peers.

(4) Reading, Grades 2-12. ELLs in Grades 2-12 may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in reading. The following proficiency level descriptors for reading are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction.
(A) Beginning. Beginning ELLs have little or no ability to read and understand English used in academic and social contexts. These students:

(i) read and understand the very limited recently practiced, memorized, or highly familiar English they have learned; vocabulary predominantly includes:

(I) environmental print;

(II) some very high-frequency words; and

(III) concrete words that can be represented by pictures;

(ii) read slowly, word by word;

(iii) have a very limited sense of English language structures;

(iv) comprehend predominantly isolated familiar words and phrases; comprehend some sentences in highly routine contexts or recently practiced, highly familiar text;

(v) are highly dependent on visuals and prior knowledge to derive meaning from text in English; and

(vi) are able to apply reading comprehension skills in English only when reading texts written for this level.

(B) Intermediate. Intermediate ELLs have the ability to read and understand simple, high-frequency English used in routine academic and social contexts. These students:

(i) read and understand English vocabulary on a somewhat wider range of topics and with increased depth; vocabulary predominantly includes:

(I) everyday oral language;

(II) literal meanings of common words;

(III) routine academic language and terms; and

(IV) commonly used abstract language such as terms used to describe basic feelings;

(ii) often read slowly and in short phrases; may re-read to clarify meaning;

(iii) have a growing understanding of basic, routinely used English language structures;

(iv) understand simple sentences in short, connected texts, but are dependent on visual cues, topic familiarity, prior knowledge, pretaught topic-related vocabulary, story predictability, and teacher/peer assistance to sustain comprehension;

(v) struggle to independently read and understand grade-level texts; and

(vi) are able to apply basic and some higher-order comprehension skills when reading texts that are linguistically accommodated and/or simplified for this level.

(C) Advanced. Advanced ELLs have the ability to read and understand, with second language acquisition support, grade-appropriate English used in academic and social contexts. These students:

(i) read and understand, with second language acquisition support, a variety of grade-appropriate English vocabulary used in social and academic contexts:

(I) with second language acquisition support, read and understand grade-appropriate concrete and abstract vocabulary, but have difficulty with less commonly encountered words;
(II) demonstrate an emerging ability to understand words and phrases beyond their literal meaning; and

(III) understand multiple meanings of commonly used words;

(ii) read longer phrases and simple sentences from familiar text with appropriate rate and speed;

(iii) are developing skill in using their growing familiarity with English language structures to construct meaning of grade-appropriate text; and

(iv) are able to apply basic and higher-order comprehension skills when reading grade-appropriate text, but are still occasionally dependent on visuals, teacher/peer assistance, and other linguistically accommodated text features to determine or clarify meaning, particularly with unfamiliar topics.

(D) Advanced high. Advanced high ELLs have the ability to read and understand, with minimal second language acquisition support, grade-appropriate English used in academic and social contexts. These students:

(i) read and understand vocabulary at a level nearly comparable to that of their native English-speaking peers, with some exceptions when low-frequency or specialized vocabulary is used;

(ii) generally read grade-appropriate, familiar text with appropriate rate, speed, intonation, and expression;

(iii) are able to, at a level nearly comparable to native English-speaking peers, use their familiarity with English language structures to construct meaning of grade-appropriate text; and

(iv) are able to apply, with minimal second language acquisition support and at a level nearly comparable to native English-speaking peers, basic and higher-order comprehension skills when reading grade-appropriate text.

(5) Writing, Kindergarten-Grade 1. ELLs in Kindergarten and Grade 1 may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in writing. The following proficiency level descriptors for writing are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction and should take into account developmental stages of emergent writers.

(A) Beginning. Beginning ELLs have little or no ability to use the English language to build foundational writing skills. These students:

(i) are unable to use English to explain self-generated writing such as stories they have created or other personal expressions, including emergent forms of writing (pictures, letter-like forms, mock words, scribbling, etc.);

(ii) know too little English to participate meaningfully in grade-appropriate shared writing activities using the English language;

(iii) cannot express themselves meaningfully in self-generated, connected written text in English beyond the level of high-frequency, concrete words, phrases, or short sentences that have been recently practiced and/or memorized; and

(iv) may demonstrate little or no awareness of English print conventions.

(B) Intermediate. Intermediate ELLs have a limited ability to use the English language to build foundational writing skills. These students:
(i) know enough English to explain briefly and simply self-generated writing, including emergent forms of writing, as long as the topic is highly familiar and concrete and requires very high-frequency English;

(ii) can participate meaningfully in grade-appropriate shared writing activities using the English language only when the writing topic is highly familiar and concrete and requires very high-frequency English;

(iii) express themselves meaningfully in self-generated, connected written text in English when their writing is limited to short sentences featuring simple, concrete English used frequently in class; and

(iv) frequently exhibit features of their primary language when writing in English such as primary language words, spelling patterns, word order, and literal translating.

(C) Advanced. Advanced ELLs have the ability to use the English language to build, with second language acquisition support, foundational writing skills. These students:

(i) use predominantly grade-appropriate English to explain, in some detail, most self-generated writing, including emergent forms of writing;

(ii) can participate meaningfully, with second language acquisition support, in most grade-appropriate shared writing activities using the English language;

(iii) although second language acquisition support is needed, have an emerging ability to express themselves in self-generated, connected written text in English in a grade-appropriate manner; and

(iv) occasionally exhibit second language acquisition errors when writing in English.

(D) Advanced high. Advanced high ELLs have the ability to use the English language to build, with minimal second language acquisition support, foundational writing skills. These students:

(i) use English at a level of complexity and detail nearly comparable to that of native English-speaking peers when explaining self-generated writing, including emergent forms of writing;

(ii) can participate meaningfully in most grade-appropriate shared writing activities using the English language; and

(iii) although minimal second language acquisition support may be needed, express themselves in self-generated, connected written text in English in a manner nearly comparable to their native English-speaking peers.

(6) Writing, Grades 2-12. ELLs in Grades 2-12 may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in writing. The following proficiency level descriptors for writing are sufficient to describe the overall English language proficiency levels of ELLs in this language domain in order to linguistically accommodate their instruction.

(A) Beginning. Beginning ELLs lack the English vocabulary and grasp of English language structures necessary to address grade-appropriate writing tasks meaningfully. These students:

(i) have little or no ability to use the English language to express ideas in writing and engage meaningfully in grade-appropriate writing assignments in content area instruction;

(ii) lack the English necessary to develop or demonstrate elements of grade-appropriate writing such as focus and coherence, conventions, organization, voice, and development of ideas in English; and

(iii) exhibit writing features typical at this level, including:
(I) ability to label, list, and copy;

(II) high-frequency words/phrases and short, simple sentences (or even short paragraphs) based primarily on recently practiced, memorized, or highly familiar material; this type of writing may be quite accurate;

(III) present tense used primarily; and

(IV) frequent primary language features (spelling patterns, word order, literal translations, and words from the student’s primary language) and other errors associated with second language acquisition may significantly hinder or prevent understanding, even for individuals accustomed to the writing of ELLs.

(B) Intermediate. Intermediate ELLs have enough English vocabulary and enough grasp of English language structures to address grade-appropriate writing tasks in a limited way. These students:

(i) have a limited ability to use the English language to express ideas in writing and engage meaningfully in grade-appropriate writing assignments in content area instruction;

(ii) are limited in their ability to develop or demonstrate elements of grade-appropriate writing in English; communicate best when topics are highly familiar and concrete, and require simple, high-frequency English; and

(iii) exhibit writing features typical at this level, including:

(I) simple, original messages consisting of short, simple sentences; frequent inaccuracies occur when creating or taking risks beyond familiar English;

(II) high-frequency vocabulary; academic writing often has an oral tone;

(III) loosely connected text with limited use of cohesive devices or repetitive use, which may cause gaps in meaning;

(IV) repetition of ideas due to lack of vocabulary and language structures;

(V) present tense used most accurately; simple future and past tenses, if attempted, are used inconsistently or with frequent inaccuracies;

(VI) undetailed descriptions, explanations, and narrations; difficulty expressing abstract ideas;

(VII) primary language features and errors associated with second language acquisition may be frequent; and

(VIII) some writing may be understood only by individuals accustomed to the writing of ELLs; parts of the writing may be hard to understand even for individuals accustomed to ELL writing.

(C) Advanced. Advanced ELLs have enough English vocabulary and command of English language structures to address grade-appropriate writing tasks, although second language acquisition support is needed. These students:

(i) are able to use the English language, with second language acquisition support, to express ideas in writing and engage meaningfully in grade-appropriate writing assignments in content area instruction;

(ii) know enough English to be able to develop or demonstrate elements of grade-appropriate writing in English, although second language acquisition support is particularly needed when topics are abstract, academically challenging, or unfamiliar; and

(iii) exhibit writing features typical at this level, including:
(I) grasp of basic verbs, tenses, grammar features, and sentence patterns; partial grasp of more complex verbs, tenses, grammar features, and sentence patterns;

(II) emerging grade-appropriate vocabulary; academic writing has a more academic tone;

(III) use of a variety of common cohesive devices, although some redundancy may occur;

(IV) narrations, explanations, and descriptions developed in some detail with emerging clarity; quality or quantity declines when abstract ideas are expressed, academic demands are high, or low-frequency vocabulary is required;

(V) occasional second language acquisition errors; and

(VI) communications are usually understood by individuals not accustomed to the writing of ELLs.

(D) Advanced high. Advanced high ELLs have acquired the English vocabulary and command of English language structures necessary to address grade-appropriate writing tasks with minimal second language acquisition support. These students:

(i) are able to use the English language, with minimal second language acquisition support, to express ideas in writing and engage meaningfully in grade-appropriate writing assignments in content area instruction;

(ii) know enough English to be able to develop or demonstrate, with minimal second language acquisition support, elements of grade-appropriate writing in English; and

(iii) exhibit writing features typical at this level, including:

(I) nearly comparable to writing of native English-speaking peers in clarity and precision with regard to English vocabulary and language structures, with occasional exceptions when writing about academically complex ideas, abstract ideas, or topics requiring low-frequency vocabulary;

(II) occasional difficulty with naturalness of phrasing and expression; and

(III) errors associated with second language acquisition are minor and usually limited to low-frequency words and structures; errors rarely interfere with communication.

(e) Effective date. The provisions of this section supersede the ESL standards specified in Chapter 128 of this title (relating to Texas Essential Knowledge and Skills for Spanish Language Arts and English as a Second Language) upon the effective date of this section.

Source: The provisions of this §74.4 adopted to be effective December 25, 2007, 32 TexReg 9615.